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# The Topological Group of Autohomeomorphisms of Homogeneous Functionally Alexandroff Spaces

by

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# THE TOPOLOGICAL GROUP OF AUTOHOMEOMORPHISMS OF HOMOGENEOUS FUNCTIONALLY ALEXANDROFF SPACES

SAMI LAZAAR AND HOUSSEM SABRI

Abstract. In this paper, we study topological properties of the group of autohomeomorphisms of homogeneous functionally Alexandroff spaces.

In particular, when the functionally Alexandroff space $(X, \mathcal{P}(f))$ is a connected homogeneous space where the map $f$ is $\kappa$-to-one for some finite cardinal number $\kappa > 1$, we prove that its autohomeomorphism group $\mathcal{H}(X)$ equipped with the permutation topology is a second countable, totally disconnected, locally compact (s.c.t.d.l.c.), non discrete Hausdorff group. Furthermore, in this case, or even if $\kappa$ is an infinite cardinal number, we prove that the topology of pointwise convergence, the compact-open topology and the permutation topology which equip the topological group $\mathcal{H}(X)$ are equivalents.

Finally, one of the main results of this paper is to prove that the group $\mathcal{H}(X)$ is not locally compact when $\kappa$ is an infinite cardinal number. Consequently, we deduce that the group of all isometries of a regular tree is locally compact if and only if the tree is locally finite.

## 1. Introduction

A topological space $X$ is said to be Alexandroff space if any arbitrary intersection of open sets is open. They have been introduced in 1937 by P. S. Alexandrov in [3] under the name Diskrete Räume "discrete space".

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