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TOPOLOGICAL CONJUGACY FOR THE MORSE MINIMAL SYSTEM: AN EXAMPLE

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Abstract. We give an example of a subshift that is topologically conjugate to the Morse minimal system but is not generated by a constant-length substitution. This shows that the property of being generated by a constant-length substitution is not a topological conjugacy invariant. Our proof illustrates the usefulness of techniques developed by Coven, Dekking, and Keane [3].

1. Introduction and Preliminaries

Given a finite alphabet $\mathcal{A}$, let $\mathcal{A}^{\mathbb{Z}}$ denote the set of doubly-infinite sequences $x = (x_i)_{i\in\mathbb{Z}}$ on $\mathcal{A}$, and let $\sigma : \mathcal{A}^{\mathbb{Z}} \to \mathcal{A}^{\mathbb{Z}}$ be the left shift: $\sigma(x)_i = x_{i+1}$. If a subset $X \subseteq \mathcal{A}^{\mathbb{Z}}$ is σ-invariant and closed in the product topology, then the pair (X, σ) is a *subshift*. The subshift is *minimal* if $X = \overline{\{\sigma^n(x) : n \in \mathbb{Z}\}}$ for every $x \in X$. Given another subshift (Y, σ), a continuous function $\psi : X \to Y$ such that $\psi \circ \sigma = \sigma \circ \psi$ is called a *factor map* if it is surjective, and a *topological conjugacy* if it is a homeomorphism. For more background on subshifts, see [10].

Given an integer $L \geq 2$, a *substitution* of constant length L is a mapping $\theta : \mathcal{A} \to \mathcal{A}^L$. Higher powers $\theta^k : \mathcal{A} \to \mathcal{A}^{L^k}$ are defined recursively: if $\theta(a) = a_1 \cdots a_\ell$, then $\theta^2(a) := \theta(a_1)\cdots\theta(a_\ell)$; $\theta^3(a) := \theta^2(a_1)\cdots\theta^2(a_\ell)$; and so on. The substitution is *primitive* if, for all $a, b \in \mathcal{A}$, there exists $k \in \mathbb{N}$ such that $\theta^k(a)$ contains b. It is well-known that a primitive

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