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by

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PRESERVATION OF A NEIGHBORHOOD BASE OF A DISCRETE SET BY FORCING

AKIRA IWASA

Abstract. Let $\langle X, \tau \rangle$ be a topological space and let A be a discrete subset of X. We study under what circumstances a neighborhood base of A remains a neighborhood base of A in forcing extensions.

1. Introduction

In [3], we studied the preservation of a neighborhood base of a set by ccc forcings. This note is a sequel to [3] and this time we consider the case where sets are *discrete*; that is, we study under what circumstances a neighborhood base of a discrete set remains a neighborhood base of the set in forcing extensions. Let us repeat some definitions from [3].

Definition 1.1. For a ground model $\mathbf{V}$ and a forcing $\mathbb{P}$, $\mathbf{V}^{\mathbb{P}}$ is the forcing extension of $\mathbf{V}$ by the forcing $\mathbb{P}$. For a topological space $\langle X, \tau \rangle$ in $\mathbf{V}$, $\langle X, \tau^{\mathbb{P}} \rangle$ is the topological space in $\mathbf{V}^{\mathbb{P}}$ such that $\tau^{\mathbb{P}}$ is the topology on X generated by τ in $\mathbf{V}^{\mathbb{P}}$.

Definition 1.2. (1) Let $\langle X, \tau \rangle$ be a topological space and let $A \subseteq X$. We denote the set of all neighborhoods of A by

$$\mathcal{N}_\tau(A) := \{H \subseteq X : (\exists U \in \tau)(A \subseteq U \subseteq H)\}.$$

If $A = \{x\}$, then we write $\mathcal{N}_\tau(x)$ for $\mathcal{N}_\tau(\{x\})$.

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