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Asymmetrics, Coherence, and Metrization

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ASYMMETRICS, COHERENCE, AND METRIZATION

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Abstract. There is Niemytzki, the fountain-head of all classical metrization results, and there is Collins-Reed-Roscoe-Rudin. Neither one implies the other. In response to Nagata's call for unification, we provide here a theorem to which both are corollaries.

In [11] Professor Nagata registered his "surprise" over the metrization condition of Collins-Reed-Roscoe-Rudin (CRRR): T_1-spaces with an open point-network are metrizable [2], [4]. (We adopt here Balogh's terminology in preference to the designation "open decreasing (G)," the latter being so tentative). In CRRR, conspicuously absent is any indication of uniformity, a *sine qua non* among the "classical" results. Professor Nagata suspected something fundamental was setting apart CRRR from the "classical" conditions. And yet he asked for unification.

These "classical" conditions are all topological realizations [6], [7] of Niemytzki's coherent symmetrics [14] and the uniformity among them is there to account for the symmetry of the symmetric. On the other hand, CRRR is not and there is no need for it to have any uniformity.

To have unification, we need to banish the symmetry in the symmetric of Niemytzki's and study coherence anew in the context of what is to be called an asymmetric. Of our results, Corollary 3.1.1 accounts for, Theorem 3.1 improves upon, while Corollary 3.1.3 and Theorem 3.2 supplement CRRR. Corollary 3.1.4 is clearly an improvement on Niemytzki. Thus our Theorem 3.1 unites the "classical" results via Niemytzki on the one hand and CRRR on the other as its corollaries, as Professor Nagata wished. Apparently, efforts to derive CRRR from Niemytzki came to nil [11]. Neither does Niemytzki follow from CRRR, in spite of what Balogh [2] says. It is in this context that Theorem 3.1 is important.

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