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ON THE PARACOMPACTNESS OF A CLASS OF MONOTONICALLY NORMAL SPACES

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Abstract. We prove that if a monotonically normal space X is both a weak $P(\lambda)$-space and a λ-perfect space, then X is paracompact. As a corollary, we obtain a new proof that linearly stratifiable spaces are paracompact. The main tool in our proofs is the Balogh–Rudin theorem that says a monotonically normal space is paracompact if and only if it has no closed subsets homeomorphic to a stationary subset of some regular uncountable cardinal κ.

1. Introduction

Jack C. Ceder [3] originally defined a stratifiable space to be a T_1-space with a σ-cushioned pair-base. It is more common, however, to use as the definition the following characterization of Carlos J. R. Borges [2] (or its complementary form in terms of closed sets). We state the definition for all linearly stratifiable spaces.

Definition 1.1 ([11], c.f. [2])**.** A T_1-space $(X, \mathcal{T})$ is said to be *stratifiable over* λ, where λ is an infinite cardinal number, provided there exists a function $S : \lambda \times \mathcal{T} \to \mathcal{T}$ satisfying for all $\beta < \lambda$ and $U \in \mathcal{T}$,

LS_1**:** $cl_X[S(\beta, U)] \subset U$;
LS_2**:** $\bigcup\{S(\beta, U) : \beta < \lambda\} = U$;
LS_3**:** if $U, W \in \mathcal{T}$ and $U \subset W$, then $S(\beta, U) \subset S(\beta, W)$;
LS_4**:** if $\gamma < \beta < \lambda$, then $S(\gamma, U) \subset S(\beta, U)$.

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