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TOPOLOGY PROCEEDINGS

Volume 56, 2020

Pages 111–124

http://topology.nipissingu.ca/tp/

Negatively Curved Codimension One Distributions

by

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Electronically published on September 24, 2019

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

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Volume 56 (2020)
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E-Published on September 24, 2019

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BARRY MINEMYER

Abstract. We consider finite volume manifold pairs (M, N) modeled on $(\mathbb{CH}^n, \mathbb{CH}^{n-2})$ and prove the existence of a special Riemannian metric g on $M \setminus N$. This metric g is complete, has finite volume, and is negatively curved when restricted to a specific non-integrable codimension one distribution $\mathscr{D}$. The existence of this metric g shows that some recent results by Grigori Avramidi and T. Tam Nguyen Phan [*Half dimensional collapse of ends of manifolds of nonpositive curvature*. To appear in Geom. Funct. Anal.] cannot, in some sense, be extended to distributions on manifolds.

1. Introduction

Let $\mathbb{CH}^n$ denote n-dimensional complex hyperbolic space. If M is a Riemannian manifold and N a totally geodesic submanifold of M, we say that the pair (M, N) is *modeled on* $(\mathbb{CH}^n, \mathbb{CH}^k)$ if there exist lattices $\Gamma \subset \mathrm{Isom}(\mathbb{CH}^n)$ and $\Lambda \subset \mathrm{Isom}(\mathbb{CH}^k)$ such that $M = \mathbb{CH}^n/\Gamma$, $N = \mathbb{CH}^k/\Lambda$, and $\Lambda < \Gamma$. We also allow for the possibility that N is disconnected. That is, we allow for multiple lattices $\Lambda < \Gamma$ which correspond to different (disjoint) copies of $\mathbb{CH}^k \subset \mathbb{CH}^n$. The main result of this paper is the existence of a special metric on $M \setminus N$ when $k = n - 2$ (that is, when N has real codimension 4).

Theorem 1.1. *Suppose (M, N) is modeled on $(\mathbb{CH}^n, \mathbb{CH}^{n-2})$ with M having finite volume. Then there exists a Riemannian metric g on $M \setminus$*

2010 *Mathematics Subject Classification.* Primary 53C20, 53C21; Secondary 53C35, 57R25.

Key words and phrases. aspherical manifold, complex hyperbolic space, curvature formulas, distribution, negative sectional curvature, totally geodesic submanifold.