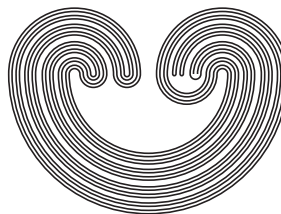


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## QUANTIZATION FOR UNIFORM DISTRIBUTIONS OF CANTOR DUSTS ON $\mathbb{R}^2$

by

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## QUANTIZATION FOR UNIFORM DISTRIBUTIONS OF CANTOR DUSTS ON $\mathbb{R}^2$

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**ABSTRACT.** Let  $P$  be a Borel probability measure on  $\mathbb{R}^2$  supported by the Cantor dusts generated by a set of  $4^u$ ,  $u \geq 1$ , contractive similarity mappings satisfying the strong separation condition. For this probability measure, we determine the optimal sets of  $n$ -means and the  $n^{\text{th}}$  quantization errors for all  $n \geq 2$ . In addition, it is shown that though the quantization dimension of the measure  $P$  is known, the quantization coefficient for  $P$  does not exist.

### 1. INTRODUCTION

Quantization for a probability distribution is the process of estimating it by a discrete probability that assumes only a finite number of levels in its support. For an in-depth analysis of quantization of probability measures, one may consult the excellent source by Sigfried Graf and Harald Luschgy [7] and the sources [1], [8], [9], [10], and [15], to name a few. In this paper, we are interested in the quantization of a particular type of continuous singular self-similar probability measures.

Let  $\mathbb{R}^d$  denote the  $d$ -dimensional Euclidean space with the Euclidean norm  $\|\cdot\|$ . For any  $d \geq 1$  and  $n \in \mathbb{N}$ , the  $n^{\text{th}}$  *quantization error* for a Borel probability measure  $P$  on  $\mathbb{R}^d$  is defined by

$$V_n := V_n(P) = \inf \left\{ \int \min_{a \in \alpha} \|x - a\|^2 dP(x) : \alpha \subset \mathbb{R}^d, \text{card}(\alpha) \leq n \right\}.$$

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*Key words and phrases.* optimal quantizer, probability measure, quantization dimension, quantization error, Sierpiński carpet.

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