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by

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QUANTIZATION FOR UNIFORM DISTRIBUTIONS OF CANTOR DUSTS ON $\mathbb{R}^2$

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ABSTRACT. Let P be a Borel probability measure on $\mathbb{R}^2$ supported by the Cantor dusts generated by a set of 4^u, $u \geq 1$, contractive similarity mappings satisfying the strong separation condition. For this probability measure, we determine the optimal sets of n-means and the n^{th} quantization errors for all $n \geq 2$. In addition, it is shown that though the quantization dimension of the measure P is known, the quantization coefficient for P does not exist.

1. INTRODUCTION

Quantization for a probability distribution is the process of estimating it by a discrete probability that assumes only a finite number of levels in its support. For an in-depth analysis of quantization of probability measures, one may consult the excellent source by Sigfried Graf and Harald Luschgy [7] and the sources [1], [8], [9], [10], and [15], to name a few. In this paper, we are interested in the quantization of a particular type of continuous singular self-similar probability measures.

Let $\mathbb{R}^d$ denote the d-dimensional Euclidean space with the Euclidean norm $\| \ \|$. For any $d \geq 1$ and $n \in \mathbb{N}$, the n^{th} *quantization error* for a Borel probability measure P on $\mathbb{R}^d$ is defined by

$$V_n := V_n(P) = \inf \Big\{ \int \min_{a \in \alpha} \|x - a\|^2 dP(x) : \alpha \subset \mathbb{R}^d, \ \text{card}(\alpha) \leq n \Big\}.$$

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