http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 56, 2020

Pages 219–236

http://topology.nipissingu.ca/tp/

A Characterization of Paracompactness of Lexicographic Products

by

Yasushi Hirata and Nobuyuki Kemoto

Electronically published on December 10, 2019

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 56 (2020)
Pages 219-236

E-Published on December 10, 2019

A CHARACTERIZATION OF PARACOMPACTNESS OF LEXICOGRAPHIC PRODUCTS

YASUSHI HIRATA AND NOBUYUKI KEMOTO

Abstract. It is known that lexicographic products of paracompact LOTS's are also paracompact; see M. J. Faber [*Metrizability in Generalized Ordered Spaces*, Mathematical Centre Tracts, No. 53. Amsterdam: Mathematisch Centrum, 1974]. After then the notion of the lexicographic products of GO-spaces is defined and the result above is extended for lexicographic products of GO-spaces and it is asked, When are lexicographic products of GO-spaces paracompact? (See Nobuyuki Kemoto, [Topology Appl., 232 (2017) pp. 267–280]). For this question, paracompactness of lexicographic products of some special cases below are characterized by Kemoto [Topology Appl. 240 (2018), 35–58]:

- lexicographic products of two GO-spaces,
- lexicographic products of any length of ordinal subspaces.

In this paper, we give a complete answer to the question above asked by Kemoto [Topology Appl., 232 (2017) pp. 267–280].

1. Introduction

All spaces are assumed to be regular T_1 and when we consider a product $\prod_{\alpha<\gamma} X_\alpha$, all X_α are assumed to have cardinality at least 2 with $\gamma \geq 2$. Set theoretical and topological terminology follows [5] and [1].

A linearly ordered set $\langle L, <_L \rangle$ has a natural topology λ_L, which is called an *interval topology*, generated by $\{(\leftarrow, x)_L : x \in L\} \cup \{(x, \rightarrow)_L : x \in L\}$ as a subbase, where $(x, \rightarrow)_L = \{z \in L : x <_L z\}$, $(x, y)_L = \{z \in L : x <_L z <_L y\}$, $(x, y]_L = \{z \in L : x <_L z \leq_L y\}$, and so on. The triple $\langle L, <_L, \lambda_L \rangle$, which is simply denoted by L, is called a *LOTS*.

2010 *Mathematics Subject Classification.* Primary 54F05, 54B10, 54B05; Secondary 54C05.

Key words and phrases. GO-space, lexicographic product, LOTS, paracompact.