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ON THE CONTINUITY PROPERTY OF THE EXACT HOMOLOGY THEORIES

LEONARD MDZINARISHVILI

Abstract. It is well known [Samuel Eilenberg and Norman Steenrod, *Foundations of Algebraic Topology.* Princeton, New Jersey: Princeton University Press, 1952] that on the category $\mathcal{K}_c$ of compact pairs and continuous maps there is a continuity property of the partially exact homology [*Foundations of Algebraic Topology*, Definition 2.3.X]. Namely, if the compact pair (X, A) is an inverse limit of the compact pairs (X_α, A_α), then the partially exact homology H_* of (X, A) is an inverse limit of homology groups of (X_α, A_α); i.e., there is an isomorphism

$$H_*(X, A) \xrightarrow{\sim} \varprojlim H_*(X_\alpha, A_\alpha).$$

It has been shown that among all the partially exact theories on the category $\mathcal{K}_C$, the Čech theory is essentially the only one satisfying this continuity axiom [*Foundations of Algebraic Topology*, Theorem 3.1.X].

We define a continuity property of the exact homology theories on the category $\mathcal{K}_C$ and prove that the homology theory on the category $\mathcal{K}_C$, satisfying all the Eilenberg–Steenrod axioms and the continuity property of the exact homology theories, exists.

Let $\mathcal{K}_C$ be the category of compact pairs (X, A) and continuous maps; let H_* be an exact homology theory. Let $\{(X_\alpha, A_\alpha)\}$ be an inverse system of compact pairs (X_α, A_α) and $(X, A) = \varprojlim (X_\alpha, A_\alpha)$. The inverse system $\{(X_\alpha, A_\alpha)\}$ generates an inverse system $\{H_*(X_\alpha, A_\alpha)\}$ and the projection $\pi_\alpha : (X, A) \to (X_\alpha, A_\alpha)$ induces the homomorphism $\pi_{\alpha,*} : H_*(X, A) \to$

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