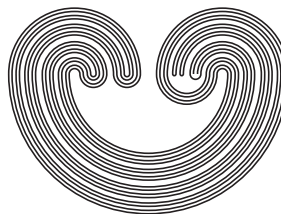


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## TOPOLOGIES GENERATED BY FAMILIES OF SETS AND STRONG POSET MODELS

by

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## TOPOLOGIES GENERATED BY FAMILIES OF SETS AND STRONG POSET MODELS

DONGSHENG ZHAO, XIAOYONG XI, AND YIXIANG CHEN

**ABSTRACT.** A poset model of a topological space  $X$  is a poset  $P$  such that  $X$  is homeomorphic to the maximal point space of  $P$  (the set  $\text{Max}(P)$  of all maximal points of  $P$  equipped with the relative Scott topology of  $P$ ). The poset models of topological spaces based on other topologies, such as Lawson topology and lower topology, have also been investigated by other people. These models establish various types of new links between posets and topological spaces. In this paper, we introduce the strong Scott topology on a poset and use it to define the strong poset model: A strong poset model of a space  $X$  is a poset  $P$  such that  $\text{Max}(P)$  (equipped with the relative strong Scott topology) is homeomorphic to  $X$ . The main aim is to establish a characterization of  $T_1$  spaces with T-generated topologies (such as the Hausdorff  $k$ -spaces) in terms of maximal point spaces of posets. A poset  $P$  is called *ME-separated* if for any elements  $x$  and  $y$  of  $P$ ,  $x \leq y$  if and only if  $\uparrow y \cap \text{Max}(P) \subseteq \uparrow x \cap \text{Max}(P)$ . We consider the topological spaces that have an ME-separated strong poset model. The main result is that a  $T_1$  space has an ME-separated strong poset model if and only if its topology is T-generated. The class of spaces whose topologies are T-generated include all Scott spaces and all Hausdorff  $k$ -spaces.

A poset model of a topological space  $X$  is a poset  $P$  such that the subspace  $\text{Max}(P)$  of all maximal points of  $P$  of the Scott space  $\Sigma P$  is homeomorphic to  $X$ . It has been proved by several authors that a topological space has a poset model if and only if it is a  $T_1$  space (see [1], [3],

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*Key words and phrases.* maximal point space; ME-separated poset; strong poset model; strong Scott topology; T-generated space.

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