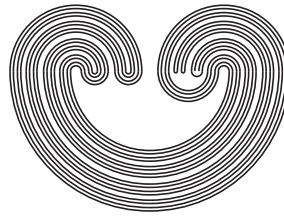


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CUSP TRANSITIVITY IN HYPERBOLIC 3-MANIFOLDS

by

ROGER VOGELER

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Department of Mathematics & Statistics
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ROGER VOGELER

ABSTRACT. Let M be a cusped finite-volume hyperbolic three-manifold with isometry group G . Then G induces a k -transitive action by permutation on the cusps of M for some integer $k \geq 0$. Generically G is trivial and $k = 0$, but $k > 0$ does occur in special cases. We show examples with $k = 1, 2, 4$. An interesting question concerns the possible number of cusps for a fixed k . Our main result provides an answer for $k = 2$ by constructing a family of manifolds having no upper bound on the number of cusps.

1. INTRODUCTION

An action of a group G on a set S is called k -transitive if, for every choice of distinct elements $x_1, \dots, x_k \in S$ and every choice of distinct targets $y_1, \dots, y_k \in S$, there is an element $g \in G$ such that $g(x_i) = y_i$. The term *transitive* means 1-transitive; actions with $k > 1$ are *multiply transitive*. The number of elements in S is the *degree*. Transitive actions are common (for example, every group acts transitively on itself by left multiplication), while multiply-transitive actions are relatively rare. The theory is well developed; see [4].

It is obvious that the isometry group of a complete finite-volume hyperbolic three-manifold induces a permutation action on the set of cusps. In this paper we call such a manifold *k-transitive* if the induced action is k -transitive. Note that this definition is of the ‘inclusive hierarchy’ type. For instance, a 3-transitive manifold is automatically 2-transitive, and possibly 4-transitive as well.

Kojima [8] shows by construction that every finite group G occurs as the isometry group of some closed hyperbolic 3-manifold. At one stage,

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