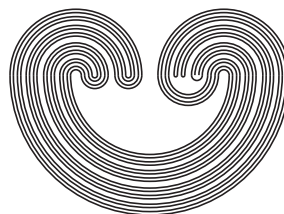


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SELECTIVITY PROPERTIES OF SPACES

by

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SELECTIVITY PROPERTIES OF SPACES

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ABSTRACT. This paper addresses several questions of Feng, Gruenhage, and Shen which arose from Michael's theory of continuous selections from countable spaces. We construct an example of a space which is $(\omega + 1)$ -selective but not \mathbb{Q} -selective from $\mathfrak{d} = \omega_1$, and an $(\omega + 1)$ -selective space which is not selective for a P -point ultrafilter from the assumption of CH. We also produce ZFC examples of Fréchet spaces where countable subsets are first countable which are not $(\omega + 1)$ -selective.

1. INTRODUCTION

Suppose X, Y are topological spaces and $\varphi : Y \rightarrow \mathcal{P}(X) \setminus \{\emptyset\}$ is a map. A general question investigated in detail by Michael asks under what conditions it is possible to find a continuous $s : Y \rightarrow X$ so that $s(y) \in \varphi(y)$ for all $y \in Y$. It is natural to require some kind of continuity assumption for φ . So let $\mathcal{F}(X)$ be the set of all nonempty closed subsets of X equipped with the Vietoris topology generated by the sets

- (1) $\{A \in \mathcal{F}(X) : A \cap W \neq \emptyset\}$
- (2) $\{A \in \mathcal{F}(X) : A \subseteq W\}$

where W ranges through open subsets of X . A map $\varphi : Y \rightarrow \mathcal{F}(X)$ is called *lower semicontinuous* if it is continuous with respect to open sets of the first kind, i.e., if for every nonempty open $W \subseteq X$, the set $\{y \in Y : \varphi(y) \cap W \neq \emptyset\}$ is open in Y .

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