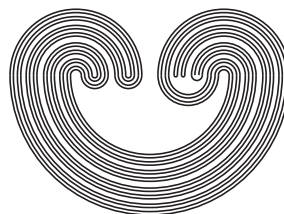


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 57, 2021

Pages 137–148

---

<http://topology.nipissingu.ca/tp/>

## COARSE SPACES, ULTRAFILTERS AND DYNAMICAL SYSTEMS

by

IGOR PROTASOV

Electronically published on July 12, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings  
Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## COARSE SPACES, ULTRAFILTERS AND DYNAMICAL SYSTEMS

IGOR PROTASOV

**ABSTRACT.** For a coarse space  $(X, \mathcal{E})$ ,  $X^\sharp$  denotes the set of all unbounded ultrafilters on  $X$  endowed with the parallelity relation:  $p \parallel q$  if there exists  $E \in \mathcal{E}$  such that  $E[P] \in q$  for each  $P \in p$ . If  $(X, \mathcal{E})$  is finitary then there exists a group  $G$  of permutations of  $X$  such that the coarse structure  $\mathcal{E}$  has the base  $\{(x, gx) : x \in X, g \in G\} : F \in [G]^{<\omega}, id \in F\}$ . We survey and analyze interplays between  $(X, \mathcal{E})$ ,  $X^\sharp$  and the dynamical system  $(G, X^\sharp)$ .

The dynamical Švarc-Milnor Theorem and Gromov Theorem arose at the dawn of *Geometric Group Theory*. In both cases, a group or a pair of groups act on some locally compact spaces, see [22, Chapter 1]. The Gromov coupling criterion was transformed into the powerful tool in coarse equivalences (see references in [23]), however some natural questions on the coarse equivalence of groups need more delicate combinatorial technique, see [4].

In this paper, we describe and survey the dynamical approach to coarse spaces originated in the algebra of the Stone-Ćech compactification. We identify the Stone-Ćech compactification  $\beta G$  of a discrete group  $G$  with the set of all ultrafilters on  $G$ . The left regular action  $G$  on  $G$  gives rise to the action of  $G$  on  $\beta G$  by  $(g, p) \mapsto gp, gp = \{gP : P \in p\}$ . In turn, the dynamical system  $(G, \beta G)$  induces on  $\beta G$  the structure of a right topological semigroup. The product  $pq$  of ultrafilters  $p, q$  is defined by  $A \in pq$  if and only if  $\{g \in G : g^{-1}A \in q\} \in p$ . The semigroup  $\beta G$  has a very rich algebraic structure and plenty of combinatorial applications; see nice paper [5], capital book [6] or booklet [9].

---

2020 *Mathematics Subject Classification.* 54D80, 20B35, 20F69.

*Key words and phrases.* Coarse spaces, ballenans, ultrafilters, dynamical systems.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.