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Coarse spaces, ultrafilters and dynamical systems

by

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Abstract. For a coarse space $(X,\mathcal{E})$, $X^\sharp$ denotes the set of all unbounded ultrafilters on X endowed with the parallelity relation: $p||q$ if there exists $E\in\mathcal{E}$ such that $E[P]\in q$ for each $P\in p$. If $(X,\mathcal{E})$ is finitary then there exists a group G of permutations of X such that the coarse structure $\mathcal{E}$ has the base $\{\{(x,gx): x\in X,\ g\in F\}: F\in[G]^{<\omega},\ id\in F\}$. We survey and analyze interplays between $(X,\mathcal{E})$, $X^\sharp$ and the dynamical system $(G,X^\sharp)$.

The dynamical Švarc-Milnor Theorem and Gromov Theorem arose at the dawn of *Geometric Group Theory.* In both cases, a group or a pair of groups act on some locally compact spaces, see [22, Chapter 1]. The Gromov coupling criterion was transformed into the powerful tool in coarse equivalences (see references in [23]), however some natural questions on the coarse equivalence of groups need more delicate combinatorial technique, see [4].

In this paper, we describe and survey the dynamical approach to coarse spaces originated in the algebra of the Stone-Čech compactification. We identify the Stone-Čech compactification βG of a discrete group G with the set of all ultrafilters on G. The left regular action G on G gives rise to the action of G on βG by $(g,p)\mapsto gp$, $gp=\{gP: P\in p\}$. In turn, the dynamical system $(G,\beta G)$ induces on βG the structure of a right topological semigroup. The product pq of ultrafilters p,q is defined by $A\in pq$ if and only if $\{g\in G: g^{-1}A\in q\}\in p$. The semigroup βG has a very rich algebraic structure and plenty of combinatorial applications; see nice paper [5], capital book [6] or booklet [9].

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