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by

Andrés Millán

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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FILTERS ON ω AND IRREDUNDANT FAMILIES

ANDRÉS MILLÁN

ABSTRACT. In this note we generalize a result by J. Cancino, O. Guzmán and A. Miller by proving that no irredundant family on any countable set can generate a finite intersection of prime ideals. We prove among other results that either there are at most $2^{\mathfrak{d}}$-many Q-points or there are $2^{\mathfrak{c}}$-many Rudin-Keisler incomparable Q-points. We also give another characterization of the reaping number $\mathfrak{r}$ in terms of partitions of ω.

1. Preliminaries

We will use standard set theoretic notation. If $A, B \in [\omega]^\omega$ we write $A \subseteq^* B$ provided that $A \setminus B$ is finite. Letters $\mathcal{A}$, $\mathcal{I}$ and $\mathcal{R}$ will denote families of subsets of ω. A family $\mathcal{A}$ has the strong finite intersection property provided every finite subfamily of $\mathcal{A}$ has an infinite intersection; this property will be denoted in the text as SFIP. Given a family of sets we denote $\langle \mathcal{A} \rangle$, depending on the context, either the ideal or the filter generated by $\mathcal{A}$. A family $\mathcal{G} \subseteq \omega^\omega$ is *dominating* provided for every $h \in \omega^\omega$ there is a $g \in \mathcal{G}$ such that $\forall^\infty n < \omega \ h(n) < g(n)$. Letter $\mathfrak{d}$ denotes the minimum cardinality of a dominating family on ω^ω. A family $\mathcal{R} \subseteq [\omega]^\omega$ is reaping provided for every $A \in [\omega]^\omega$ there is a $R \in \mathcal{R}$ such that either $R \subseteq^* A$ or $R \subseteq^* \omega \setminus A$. A family $I \subseteq [\omega]^\omega$ is independent provided for every $\mathcal{A}, \mathcal{B} \in [I]^{<\omega}$ with $\mathcal{A} \cap \mathcal{B} = \emptyset$

$$\left| \bigcap_{A \in \mathcal{A}} A \cap \bigcap_{B \in \mathcal{B}} (\omega \setminus B) \right| = \omega.$$

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