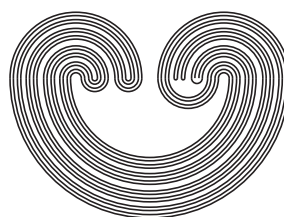


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## FILTERS ON $\omega$ AND IRREDUNDANT FAMILIES

by

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## FILTERS ON $\omega$ AND IRREDUNDANT FAMILIES

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**ABSTRACT.** In this note we generalize a result by J. Cancino, O. Guzmán and A. Miller by proving that no irredundant family on any countable set can generate a finite intersection of prime ideals. We prove among other results that either there are at most  $2^{\mathfrak{d}}$ -many  $Q$ -points or there are  $2^{\mathfrak{c}}$ -many Rudin-Keisler incomparable  $Q$ -points. We also give another characterization of the reaping number  $\mathfrak{r}$  in terms of partitions of  $\omega$ .

### 1. PRELIMINARIES

We will use standard set theoretic notation. If  $A, B \in [\omega]^\omega$  we write  $A \subseteq^* B$  provided that  $A \setminus B$  is finite. Letters  $\mathcal{A}$ ,  $\mathcal{I}$  and  $\mathcal{R}$  will denote families of subsets of  $\omega$ . A family  $\mathcal{A}$  has the strong finite intersection property provided every finite subfamily of  $\mathcal{A}$  has an infinite intersection; this property will be denoted in the text as SFIP. Given a family of sets we denote  $\langle \mathcal{A} \rangle$ , depending on the context, either the ideal or the filter generated by  $\mathcal{A}$ . A family  $\mathcal{G} \subseteq \omega^\omega$  is *dominating* provided for every  $h \in \omega^\omega$  there is a  $g \in \mathcal{G}$  such that  $\forall^\infty n < \omega$   $h(n) < g(n)$ . Letter  $\mathfrak{d}$  denotes the minimum cardinality of a dominating family on  $\omega^\omega$ . A family  $\mathcal{R} \subseteq [\omega]^\omega$  is reaping provided for every  $A \in [\omega]^\omega$  there is a  $R \in \mathcal{R}$  such that either  $R \subseteq^* A$  or  $R \subseteq^* \omega \setminus A$ . A family  $\mathcal{I} \subseteq [\omega]^\omega$  is independent provided for every  $\mathcal{A}, \mathcal{B} \in [\mathcal{I}]^{<\omega}$  with  $\mathcal{A} \cap \mathcal{B} = \emptyset$

$$\left| \bigcap_{A \in \mathcal{A}} A \cap \bigcap_{B \in \mathcal{B}} (\omega \setminus B) \right| = \omega.$$

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