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by

Takuma Imamura

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Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
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ASYMMETRIC COMPLETIONS OF PARTIAL METRIC SPACES

TAKUMA IMAMURA

Abstract. Xun Ge and Shou Lin (2015) prove the existence and the uniqueness of p-Cauchy completions of partial metric spaces under symmetric denseness. They ask if every (non-empty) partial metric space X has a p-Cauchy completion $\bar{X}$ such that X is dense but not symmetrically dense in $\bar{X}$. We construct asymmetric p-Cauchy completions for all non-empty partial metric spaces. This gives a positive answer to the question. We also provide a nonstandard construction of partial metric completions.

Introduction

Metric spaces are among the most investigated types of spaces. Whilst all metric spaces are T_1, non-T_1 spaces have also been paid attention, particularly in the context of denotational semantics of programming languages. In order to deal with such spaces in a similar fashion to metric spaces, Steven G. Matthews [8] and [9] introduces the notion of partial metric. Roughly speaking, a partial metric space is a generalised metric space where the self-distance is not necessarily zero.

Definition 0.1 ([9]). A *partial metric* (*pmetric*) on a set X is a function $p_X : X \times X \to \mathbb{R}_{\geq 0}$ that satisfies the following axioms:

(P1) $p_X(x,x) = p_X(x,y) = p_X(y,y) \implies x = y$;
(P2) $p_X(x,x) \leq p_X(x,y)$;

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