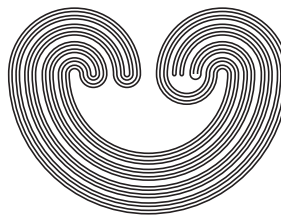


<http://topology.auburn.edu/tp/>

---

# TOPOLOGY PROCEEDINGS



Volume 58, 2021

Pages 1–12

---

<http://topology.nipissingu.ca/tp/>

## ASYMMETRIC COMPLETIONS OF PARTIAL METRIC SPACES

by

TAKUMA IMAMURA

Electronically published on March 22, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.

---

### Topology Proceedings

**Web:** <http://topology.auburn.edu/tp/>

**Mail:** Topology Proceedings

Department of Mathematics & Statistics

Auburn University, Alabama 36849, USA

**E-mail:** [topolog@auburn.edu](mailto:topolog@auburn.edu)

**ISSN:** (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

## ASYMMETRIC COMPLETIONS OF PARTIAL METRIC SPACES

TAKUMA IMAMURA

**ABSTRACT.** Xun Ge and Shou Lin (2015) prove the existence and the uniqueness of  $p$ -Cauchy completions of partial metric spaces under symmetric denseness. They ask if every (non-empty) partial metric space  $X$  has a  $p$ -Cauchy completion  $\bar{X}$  such that  $X$  is dense but not symmetrically dense in  $\bar{X}$ . We construct asymmetric  $p$ -Cauchy completions for all non-empty partial metric spaces. This gives a positive answer to the question. We also provide a nonstandard construction of partial metric completions.

### INTRODUCTION

Metric spaces are among the most investigated types of spaces. Whilst all metric spaces are  $T_1$ , non- $T_1$  spaces have also been paid attention, particularly in the context of denotational semantics of programming languages. In order to deal with such spaces in a similar fashion to metric spaces, Steven G. Matthews [8] and [9] introduces the notion of partial metric. Roughly speaking, a partial metric space is a generalised metric space where the self-distance is not necessarily zero.

**Definition 0.1** ([9]). A *partial metric* ( *$p$ metric*) on a set  $X$  is a function  $p_X: X \times X \rightarrow \mathbb{R}_{\geq 0}$  that satisfies the following axioms:

- (P1)  $p_X(x, x) = p_X(x, y) = p_X(y, y) \implies x = y$ ;
- (P2)  $p_X(x, x) \leq p_X(x, y)$ ;

---

2020 *Mathematics Subject Classification.* Primary 54E50; Secondary 54J05.

*Key words and phrases.* Cauchy completions; denseness; nonstandard analysis; partial metric spaces; symmetric denseness.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.auburn.edu/tp/subscriptioninfo.html> for information.