http://topology.auburn.edu/tp/

TOPOLOGY PROCEEDINGS

Volume 58, 2021

Pages 13–22

http://topology.nipissingu.ca/tp/

$\mathbf{\Pi}^0_2$ Subsets of Domain-Complete Spaces and Countably Correlated Spaces

by

Jean Goubault-Larrecq

Electronically published on March 24, 2020

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.

Topology Proceedings

Web:	http://topology.auburn.edu/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.

http://topology.auburn.edu/tp/
http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS
Volume 58 (2021)
Pages 13-22

E-Published on March 24, 2020

$\mathbf{\Pi}^0_2$ SUBSETS OF DOMAIN-COMPLETE SPACES AND COUNTABLY CORRELATED SPACES

JEAN GOUBAULT-LARRECQ

Abstract. We show that every $\mathbf{\Pi}^0_2$ subset of a domain-complete space is domain-complete. This implies that Ruiyuan Chen's countably correlated spaces are all domain-complete (see *Notes on quasi-Polish spaces*. Available at arXiv:1809.07440v1 [math.LO]).

1. Introduction

A *domain-complete space* is a space homeomorphic to a G_δ subset of a continuous dcpo. Those spaces were introduced in [4] and contain all continuous dcpos; all of Matthew de Brecht's quasi-Polish spaces [3], in particular, all Polish spaces; all continuous complete quasi-metric spaces in their d-Scott topology; and, in particular, all completely metrizable spaces.

The following is mentioned in [4, p. 33] as open problem (v): Is every subspace obtained as a $\mathbf{\Pi}^0_2$ subset of a domain-complete space again domain-complete? We give a positive answer to this problem here, where we also solve open problems (vi) and (vii) of the same paper, as we will see at the end of this paper.

2. Preliminaries

A *dcpo* is a poset in which every directed family D has a supremum $\sup^\uparrow D$. The *way-below* relation $\ll$ on a dcpo Y is defined by $x \ll y$ if and only if every directed family D such that $y \leq \sup^\uparrow D$ contains an element above x. The following relations hold: $x \ll y$ implies $x \leq y$; $x \leq y \ll z$

2020 *Mathematics Subject Classification.* Primary 54G99.

Key words and phrases. $\mathbf{\Pi}^0_2$ subset, countably correlated space, domain-complete space.

©2020 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See http://topology.auburn.edu/tp/subscriptioninfo.html for information.