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SMOOTH CONVEX BODIES IN $\mathbb{R}^n$ WITH DENSE UNION OF FACETS

STOYU T. BAROV

ABSTRACT. Let B be closed and convex in $\mathbb{R}^n$; B is called a *convex body* if B is compact and has a nonempty interior with respect to $\mathbb{R}^n$. In addition, B is *smooth* if B has a unique supporting hyperplane at every boundary point. Let $k, n \in \mathbb{N}$ with $k < n$ and let $\mathbb{L}_k^n$ denote the *Grassmann manifold* consisting of all k-dimensional linear subspaces in $\mathbb{R}^n$. An intersection F of B and a supporting hyperplane is called a *facet* if $\dim F = n-1$. A point x of B is called *exposed* by $\mathcal{P} \subset \mathbb{L}_k^n$ if there is a $P \in \mathcal{P}$ such that $(x+P) \cap B = \{x\}$. In this paper, for every $n \geq 2$, we have constructed symmetric smooth convex bodies $B(n)$ in $\mathbb{R}^n$ whose union of all facets is dense in the boundary of $B(n)$ and so that the set of its facets defines a dense set $\mathcal{P}$ in $\mathbb{L}_k^n$ such that the set of all points in $B(n)$ exposed by $\mathcal{P}$ is empty.

1. Introduction

Let B be convex and closed in $\mathbb{R}^n$ and let $\mathbb{L}_k^n$ denote the Grassmann manifold consisting of all k-dimensional linear subspaces of $\mathbb{R}^n$; see Definition 2.3. Let $\mathcal{P} \subset \mathbb{L}_k^n$. A point $x \in B$ is *exposed by* $\mathcal{P}$ if there is a $P \in \mathcal{P}$ such that $(x+P) \cap B = \{x\}$. In [6], the concept of an exposed point is defined, that is, a point exposed by $\mathbb{L}_{n-1}^n$. In principle, our definition generalizes that concept. By $\mathcal{X}_{\mathrm{p}}^k(B, \mathcal{P})$ we denote the set of all points in B exposed by $\mathcal{P}$. A set $C \subset \mathbb{R}^n$ is called a $\mathcal{P}$-imitation of B if $C + P = B + P$ for every $P \in \mathcal{P}$. Let $\mathcal{X}_{\mathrm{t}}^k(B, \mathcal{P}) = \bigcap\{C \subset B \colon C \text{ is a closed } \mathcal{P}\text{-imitation of } B\}$. In general, under some conditions, if $\mathcal{P} \subset \overline{\operatorname{int} \mathcal{P}}$ is not empty, then $\mathcal{X}_{\mathrm{t}}^k(B, \mathcal{P})$ contains

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