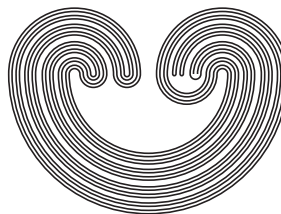


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by

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SMOOTH CONVEX BODIES IN \mathbb{R}^n WITH DENSE UNION OF FACETS

STOYU T. BAROV

ABSTRACT. Let B be closed and convex in \mathbb{R}^n ; B is called a *convex body* if B is compact and has a nonempty interior with respect to \mathbb{R}^n . In addition, B is *smooth* if B has a unique supporting hyperplane at every boundary point. Let $k, n \in \mathbb{N}$ with $k < n$ and let \mathbb{L}_k^n denote the *Grassmann manifold* consisting of all k -dimensional linear subspaces in \mathbb{R}^n . An intersection F of B and a supporting hyperplane is called a *facet* if $\dim F = n - 1$. A point x of B is called *exposed* by $\mathcal{P} \subset \mathbb{L}_k^n$ if there is a $P \in \mathcal{P}$ such that $(x + P) \cap B = \{x\}$. In this paper, for every $n \geq 2$, we have constructed symmetric smooth convex bodies $B(n)$ in \mathbb{R}^n whose union of all facets is dense in the boundary of $B(n)$ and so that the set of its facets defines a dense set \mathcal{P} in \mathbb{L}_k^n such that the set of all points in $B(n)$ exposed by \mathcal{P} is empty.

1. INTRODUCTION

Let B be convex and closed in \mathbb{R}^n and let \mathbb{L}_k^n denote the Grassmann manifold consisting of all k -dimensional linear subspaces of \mathbb{R}^n ; see Definition 2.3. Let $\mathcal{P} \subset \mathbb{L}_k^n$. A point $x \in B$ is *exposed by* \mathcal{P} if there is a $P \in \mathcal{P}$ such that $(x + P) \cap B = \{x\}$. In [6], the concept of an exposed point is defined, that is, a point exposed by \mathbb{L}_{n-1}^n . In principle, our definition generalizes that concept. By $\mathcal{X}_p^k(B, \mathcal{P})$ we denote the set of all points in B exposed by \mathcal{P} . A set $C \subset \mathbb{R}^n$ is called a \mathcal{P} -imitation of B if $C + P = B + P$ for every $P \in \mathcal{P}$. Let $\mathcal{X}_t^k(B, \mathcal{P}) = \bigcap \{C \subset B : C \text{ is a closed } \mathcal{P}\text{-imitation of } B\}$. In general, under some conditions, if $\mathcal{P} \subset \overline{\text{int } \mathcal{P}}$ is not empty, then $\mathcal{X}_t^k(B, \mathcal{P})$ contains

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