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COMPLETENESS OF LEXICOGRAPHIC PRODUCTS OF GO-SPACES

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Abstract. Variations of Dedekind completeness of lexicographic products of GO-spaces are studied. As a corollary, we see that whenever γ is limit and all GO-spaces X_α's have minimal elements but have no maximal elements, connectedness of a lexicographic product $\prod_{\alpha<\gamma} X_\alpha$ implies *non*-connectedness of all X_α's.

1. Introduction

All spaces are assumed to be regular T_1 and when we consider a product $\prod_{\alpha<\gamma} X_\alpha$, all X_α are assumed to have cardinality at least 2 with $\gamma \geq 2$. Set theoretical and topological terminology follow [9] and [1]. The following are well known:

- a LOTS X is compact if and only if every subset A of X has a least upper bound $\sup_X A$, where $\sup_X \emptyset$ is defined to be the minimal element $\min X$ of X; see [1, Problem 3.12.3(a)];
- a lexicographic product $\prod_{\alpha<\gamma} X_\alpha$ of LOTSs is compact if and only if all X_α's are compact; see [2, Theorem 4.2.1].

Obviously, a LOTS X is compact if and only if both of the following properties hold:

(a) every *non-empty* subset A of X has a least upper bound $\sup_X A$;
(b) every *non-empty* subset A of X has a greatest lower bound $\inf_X A$.

One might conjecture that if a LOTS X_α satisfies property (a) for every $\alpha < \gamma$, then the lexicographic product $\prod_{\alpha<\gamma} X_\alpha$ also satisfies property

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