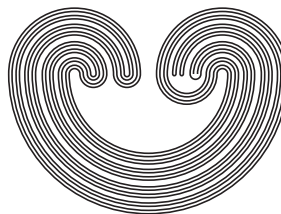


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THE KAUFFMAN BRACKET EXPANSION OF A GENERALIZED CROSSING

by

REBECCA SORSEN AND ALEXANDER ZUPAN

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Auburn University, Alabama 36849, USA

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THE KAUFFMAN BRACKET EXPANSION OF A GENERALIZED CROSSING

REBECCA SORSEN AND ALEXANDER ZUPAN

ABSTRACT. We examine the Kauffman bracket expansion of the generalized crossing Δ_n , a half-twist on n parallel strands, as an element of the Temperley–Lieb algebra with coefficients in $\mathbb{Z}[A, A^{-1}]$. In particular, we determine the minimum and maximum degrees of all possible coefficients appearing in this expansion. Our main theorem shows that the maximum such degree is quadratic in n , while the minimum such degree is linear. We also include an appendix with explicit expansions for n at most six.

1. INTRODUCTION

The Jones polynomial, introduced in [4], had a revolutionary impact on classical knot theory, fundamentally altering the fabric of low-dimensional topology. One well-known method of computing the Jones polynomial is via the Kauffman bracket $\langle \cdot \rangle$, which gives a set of rules for iteratively converting a knot diagram D into an element $\langle D \rangle$ of $\mathbb{Z}[A, A^{-1}]$ [5]. Normalizing this polynomial $\langle D \rangle$ using the writhe of D yields the Jones polynomial of K . More generally, the Kauffman bracket can also be applied to any n -stranded tangle diagram \mathcal{T} . In this case, $\langle \mathcal{T} \rangle$ is an element of the Temperley–Lieb algebra TL_n (see [7]) with coefficients in $\mathbb{Z}[A, A^{-1}]$.

The main purpose of this paper is to elicit essential characteristics of the Kauffman bracket of a *generalized crossing* Δ_n , the tangle diagram obtained by performing a half-twist on n parallel unknotted strands. As an element of the braid group, Δ_n is sometimes called the *Garside element*. In [6], Jeffrey Meier and the second author produce a family of

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