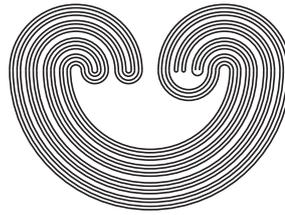


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## INTERVAL-EXPRESSED TREE-LIKE CONTINUA WITH THE FIXED POINT PROPERTY

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## INTERVAL-EXPRESSED TREE-LIKE CONTINUA WITH THE FIXED POINT PROPERTY

M. M. MARSH

**ABSTRACT.** Let  $\mathcal{T}$  be the class of tree-like continua that admit representations as inverse limits on  $[0, 1]$  with surjective upper semi-continuous set-valued functions. We show (1) if  $X \in \mathcal{T}$  with interval-valued bonding functions  $f_i$ , where there exists  $m \geq 1$  such that, for each  $i \geq m$ , the graph of  $f_i^{-1}$  contains the graph of an interval-valued function, then  $X$  has the fixed point property, and (2) if  $X \in \mathcal{T}$  with set-valued bonding functions  $f_i$ , where for each  $i \geq 1$ ,  $f_i^{-1}$  is an interval-valued function, then  $X$  is a  $\lambda$ -dendroid. We also provide an example of an indecomposable, non-arclike continuum in  $\mathcal{T}$  that has the fixed point property.

### 1. INTRODUCTION

A *compactum* is a compact metric space. A *continuum* is a connected compactum. A continuous function will be referred to as a *map* or *mapping*. A continuum  $X$  has the *fixed point property* (fpp) provided that whenever  $f$  is a self-mapping on  $X$ , there is a point  $x \in X$  such that  $f(x) = x$ . If  $\epsilon > 0$ , a mapping  $f: X \rightarrow Y$  is an  $\epsilon$ -*mapping* if  $\text{diam}(f^{-1}(y)) < \epsilon$  for each  $y \in Y$ . A continuum  $X$  is *tree-like* (*arclike*) if for each  $\epsilon > 0$ , there exists an  $\epsilon$ -mapping of  $X$  onto a tree (an arc).

Let  $\mathcal{T}$  be the class of tree-like continua that admit representations as inverse limits on  $[0, 1]$  with surjective upper semi-continuous set-valued functions. We refer to continua in  $\mathcal{T}$  as *interval-expressed tree-like continua*. This is a large class of continua that includes all arclike continua, as well as non-arclike continua that may contain any arclike continuum, or may themselves be either hereditarily decomposable or indecomposable.

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*Key words and phrases.* Fixed point property, inverse limit, tree-like,  $k$ -tail sequence with interval-valued inverse functions.

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