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TOPOLOGY PROCEEDINGS

Volume 59, 2022

Pages 51–54

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Electronically published on January 17, 2021

Topology Proceedings

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ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

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E-Published on January 17, 2021

ON ISOMETRICALLY CONTAINING MAPS FOR NONEXPANSIVE MAPS ON FINITE METRIC SPACES

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Abstract. In this paper, for each n, we construct a map $f : X \to Y$ between compact zero-dimensional metric spaces which is isometrically containing for the class of nonexpansive surjections between at most n-element metric spaces of diameter ≤ 1.

1. Introduction

Answering a question of S.D. Iliadis, A.I. Oblakova [7] proved that for each natural n there exists a compact zero-dimensional metric space containing isometric copies of every n-element metric space of diameter ≤ 1.

We shall extend this result, using a different approach, to the following effect (let us recall that nonexpansive maps between metric spaces are the ones that do not increase the distance between points).

Theorem. *For any natural number n, there exists a nonexpansive surjection $f : X \to Y$ between compact zero-dimensional metric spaces X and Y such that for each nonexpansive surjection $u : E \to F$ between at most n-element metric spaces of diameter ≤ 1, there are isometric embeddings $i_E : E \to X$, $i_F : F \to Y$ with $f \circ i_E = i_F \circ u$ and $i_E(E) = f^{-1}(i_F(F))$.*

In terminology of Iliadis [3], the map $f : X \to Y$ is isometrically containing for the class of nonexpansive surjections between at most n-element metric spaces of diameter ≤ 1.

A sketch of the construction we shall present was given in [10].

2020 *Mathematics Subject Classification.* 54E40, 54E45, 54F45.

Key words and phrases. isometry, isometrically containing mapping, compact metric spaces.