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TOPOLOGY PROCEEDINGS

Volume 59, 2022

Pages 223–241

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by

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Electronically published on June 5, 2021

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/

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Volume 59 (2022)
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INEQUALITY AND EQUALITY FOR THE EXTENT OF PRODUCTS WITH A SPECIAL FACTOR

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Abstract. For a space X, let $e(X) = \sup\{|D| : D \text{ is a closed discrete subset in } X\} \cdot \omega$, which is called the *extent* of X. First, we give some examples of a rectangular product $X \times Y$ with $e(X \times Y) > e(X) \cdot e(Y) = \omega$. Secondly, we give an equivalent condition for a given space X such that $e(X \times Y) > e(X) \cdot e(Y)$ for a certain special factor Y. Finally, we discuss when $e(X \times Y) = e(X) \cdot e(Y)$ for a product $X \times Y$ with a special factor X.

1. Introduction

For a space X, recall the cardinal function defined by

$$e(X) = \sup\{|D| : D \text{ is a closed discrete subset in } X\} \cdot \omega,$$

which is called the *extent* of X. Obviously, if X is Lindelöf or countably compact, then $e(X) = \omega$.

As is well known, the Sorgenfrey line S is a hereditarily Lindelöf space with $e(S \times S) = 2^\omega > \omega = e(S) \cdot e(S)$. So Shelah raised the following problem in 1978:

Problem 1. Are there two Lindelöf spaces X and Y such that $e(X \times Y) > 2^\omega$?

For this problem, he first proved the following.

Theorem 1.1 (Shelah [24])**.** *It is consistent that there are Lindelöf spaces X and Y with all points G_δ such that $e(X \times Y) > 2^\omega$.*

2020 *Mathematics Subject Classification.* Primary 54A25, 54B10; Secondary 54D20, 54E18.

Key words and phrases. extent, product, rectangular, almost discrete, monotonically normal, σ-space, strong σ-space, $\mathbb{DC}$-like.

This research was supported by Grant-in-Aid for Scientific Research (C) 19K03606.