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by

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STRATIFIABILITY AND THE μ-SPACE PROPERTY OF FUNCTION SPACES WITH INTERMEDIATE TOPOLOGIES

KENICHI TAMANO

Dedicated to the memory of Prof. Phillip L. Zenor

Abstract. We begin to investigate the stratifiability and the μ-space property of intermediate topologies between the toplogies of $C_p(X)$ and $C_k(X)$, for a separable metrizable space X. In particular, for the space $\mathbb{P}$ of irrational numbers, we show the following:

(1) There is a family $\mathcal{K}$ of compact sets of $\mathbb{P}$ such that $C_{\mathcal{K}}(\mathbb{P})$ is an M_1-space and the topology of $C_{\mathcal{K}}(\mathbb{P})$ is strictly between that of $C_p(X)$ and that of $C_k(\mathbb{P})$.

(2) For any nonzero natural number n, let $\mathcal{K}$ be the family of all compact sets with scattered height $< n$. Then $C_{\mathcal{K}}(\mathbb{P})$ is neither a stratifiable space nor a μ-space.

1. Introduction

All spaces are assumed to be regular T_1.

A space is a *stratifiable space*, equivalently, an *M_3-space* if it has a σ-cushioned pair base. A space is an *M_2-space* if it has a σ-closure-preserving quasi-base. Gruenhage [5] and Junnila [11] showed that a space is an M_3-space if and only if it is an M_2-space. A space is an *M_1-space* if it has a σ-closure-preserving base. A space is *F_σ-metrizable* if it is a countable union of closed metrizable subspaces. A space is a *μ-space*

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Key words and phrases. Stratifiable space, M_3-space, M_1-space, μ-space, function space, topology of pointwise convergence, compact-open topology.