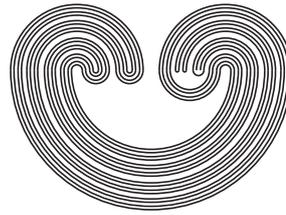


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STRATIFIABILITY AND THE  $\mu$ -SPACE PROPERTY  
OF FUNCTION SPACES  
WITH INTERMEDIATE TOPOLOGIES

by

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**STRATIFIABILITY AND THE  $\mu$ -SPACE PROPERTY  
OF FUNCTION SPACES  
WITH INTERMEDIATE TOPOLOGIES**

KENICHI TAMANO

*Dedicated to the memory of Prof. Phillip L. Zenor*

ABSTRACT. We begin to investigate the stratifiability and the  $\mu$ -space property of intermediate topologies between the topologies of  $C_p(X)$  and  $C_k(X)$ , for a separable metrizable space  $X$ . In particular, for the space  $\mathbb{P}$  of irrational numbers, we show the following:

(1) There is a family  $\mathcal{K}$  of compact sets of  $\mathbb{P}$  such that  $C_{\mathcal{K}}(\mathbb{P})$  is an  $M_1$ -space and the topology of  $C_{\mathcal{K}}(\mathbb{P})$  is strictly between that of  $C_p(X)$  and that of  $C_k(\mathbb{P})$ .

(2) For any nonzero natural number  $n$ , let  $\mathcal{K}$  be the family of all compact sets with scattered height  $< n$ . Then  $C_{\mathcal{K}}(\mathbb{P})$  is neither a stratifiable space nor a  $\mu$ -space.

**1. INTRODUCTION**

All spaces are assumed to be regular  $T_1$ .

A space is a *stratifiable space*, equivalently, an  $M_3$ -space if it has a  $\sigma$ -cushioned pair base. A space is an  $M_2$ -space if it has a  $\sigma$ -closure-preserving quasi-base. Gruenhage [5] and Junnila [11] showed that a space is an  $M_3$ -space if and only if it is an  $M_2$ -space. A space is an  $M_1$ -space if it has a  $\sigma$ -closure-preserving base. A space is  $F_\sigma$ -metrizable if it is a countable union of closed metrizable subspaces. A space is a  $\mu$ -space

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*Key words and phrases.* Stratifiable space,  $M_3$ -space,  $M_1$ -space,  $\mu$ -space, function space, topology of pointwise convergence, compact-open topology.

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