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DENSE TOPOLOGICAL GROUPS IN PAROVIČENKO SPACES

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Abstract. We show that the statement 'the Čech-Stone remainder of the discrete space ω contains a dense subspace which is (homeomorphic to) a topological group' is not a statement of ZFC. We also discuss the question of whether this result can be extended to Parovičenko spaces.

1. Introduction

All topological spaces under discussion are assumed to be Tychonoff. A space X is called a *Parovičenko space* if

(P1) X is a zero-dimensional compact space without isolated points with weight $\mathfrak{c}$,
(P2) every two disjoint open F_σ-subsets have disjoint closures, and
(P3) every nonempty G_δ in X has nonempty interior.

Moreover, X is called an F-space, [14], if each cozero-set in X is C^*-embedded in X. A normal space is an F-space if and only if X satisfies (P2), [21, 1.1.2(b)]. And, a space that satisfies (P3) is usually called an *almost P-space*.

Parovičenko [22] showed that under the Continuum Hypothesis (abbreviated: CH), every Parovičenko space is homeomorhic to ω^*, the Čech-Stone-remainder $\beta\omega \setminus \omega$ of the countable discrete space ω. In fact, CH is equivalent to the statement that every Parovičenko space is homeomorphic to ω^*, [9].

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