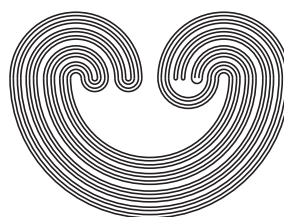


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by

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SPACES CONSTRUCTED USING \clubsuit AND WEAKER RELATED AXIOMS

PETER NYIKOS

ABSTRACT. Some very simple locally compact, first countable, Hausdorff spaces are constructed using \clubsuit and some major weakenings. The first of these examples is relevant to a wide-open problem in set-theoretic topology:

Problem. What is the least cardinality of a locally compact, first countable Hausdorff (hence Tychonoff) space that is ω_1 -compact, yet not σ -countably compact?

The first \clubsuit example is a witness to this cardinality being consistently \aleph_1 .

The other two main examples are 2-1 closed preimages of ω_1 constructed in a similar way. They illustrate how suitable \clubsuit is to show, in a simple way, that the following powerful axiom is not a consequence of ZFC.

Axiom 1. Every first countable, countably compact Hausdorff space is either compact or contains a copy of ω_1 .

A simple criterion is shown for 2-1 closed continuous preimages of ω_1 to *not* contain a copy of ω_1 : every uncountable subset must have the fibers (point-inverses) over a stationary set in its closure. Major weakenings of \clubsuit , including club guessing, are shown to be adequate to produce such spaces. A further weakening, \mathcal{U}_2 , is shown to be equivalent to a topologically defined subclass of such spaces without copies of ω_1 . These weaker axioms are independent of CH, but also compatible with major strengthenings of MA + \neg CH, and help to gauge the strength of Axiom 1, which follows from the Proper Forcing Axiom (PFA) and is also compatible with CH.

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