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SPACES CONSTRUCTED USING ♣ AND WEAKER RELATED AXIOMS

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Abstract. Some very simple locally compact, first countable, Hausdorff spaces are constructed using ♣ and some major weakenings. The first of these examples is relevant to a wide-open problem in set-theoretic topology:

Problem. What is the least cardinality of a locally compact, first countable Hausdorff (hence Tychonoff) space that is ω_1-compact, yet not σ-countably compact?

The first ♣ example is a witness to this cardinality being consistently $\aleph_1$.

The other two main examples are 2-1 closed preimages of ω_1 constructed in a similar way. They illustrate how suitable ♣ is to show, in a simple way, that the following powerful axiom is not a consequence of ZFC.

Axiom 1. Every first countable, countably compact Hausdorff space is either compact or contains a copy of ω_1.

A simple criterion is shown for 2-1 closed continuous preimages of ω_1 to *not* contain a copy of ω_1: every uncountable subset must have the fibers (point-inverses) over a stationary set in its closure. Major weakenings of ♣, including club guessing, are shown to be adequate to produce such spaces. A further weakening, $\mathfrak{U}_2$, is shown to be equivalent to a topologically defined subclass of such spaces without copies of ω_1. These weaker axioms are independent of CH, but also compatible with major strengtheings of MA + ¬CH, and help to gauge the strength of Axiom 1, which follows from the Proper Forcing Axiom (PFA) and is also compatible with CH.

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