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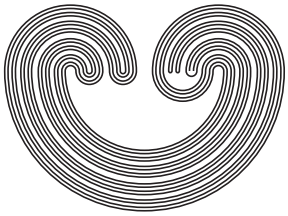

Probabilistic Powerdomains and Quasi-Continuous Domains

by

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PROBABILISTIC POWERDOMAINS AND QUASI-CONTINUOUS DOMAINS

JEAN GOUBAULT-LARRECQ

Abstract. The probabilistic powerdomain $\mathbf{V}X$ on a space X is the space of all continuous valuations on X. We show that, for every quasi-continuous domain X, $\mathbf{V}X$ is again a quasi-continuous domain, and that the Scott and weak topologies then agree on $\mathbf{V}X$. This also applies to the subspaces of probability and subprobability valuations on X, in the first case under an assumption of pointedness. We also show that the Scott and weak topologies on $\mathbf{V}X$ may differ when X is not quasi-continuous, and we give a simple, compact Hausdorff counterexample.

1. Introduction

Continuous valuations are an alternative to measures, which are popular in computer science and, notably, in the semantics of programming languages [13], [12]. The space of all continuous valuations on a topological space X is called the *probabilistic powerdomain* $\mathbf{V}X$ *on* X. It is known that the probabilistic powerdomain of a directed-complete partial order (dcpo) is a dcpo again; in short, $\mathbf{V}$ preserves dcpos. Similarly, $\mathbf{V}$ preserves continuous dcpos, but fails to preserve complete lattices and bc-domains. All of this was proved by C. Jones and G. D. Plotkin in [13], and by Jones in [12]. It is unknown whether $\mathbf{V}$ preserves RB-domains or FS-domains, except in special cases [14]. On the positive side, $\mathbf{V}$ preserves stably compact spaces [14], [3]; QRB-domains [8], [10]; and coherent quasi-continuous dcpos [19]. (The latter two results are equivalent since QRB-domains coincide with coherent quasi-continuous dcpos [17], [10], and also with Gaolin Li and Luoshan Xu's QFS-domains [18].)

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Key words and phrases. locally finitary compact space, probabilistic powerdomain, quasi-continuous domain, Scott topology, weak topology.