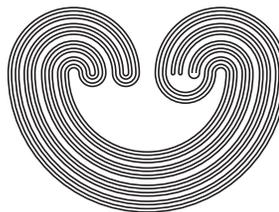


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## A SPACE WITH A LUSIN $\pi$ -BASE WHOSE SQUARE HAS NO LUSIN $\pi$ -BASE

by

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## A SPACE WITH A LUSIN $\pi$ -BASE WHOSE SQUARE HAS NO LUSIN $\pi$ -BASE

MIKHAIL PATRAKEEV

ABSTRACT. We construct a space  $X$  that has a Lusin  $\pi$ -base and such that  $X^2$  has no Lusin  $\pi$ -base.

### 1. INTRODUCTION

The class of topological spaces with a Lusin  $\pi$ -base (see Definition 2.7) was introduced in [4]; this class equals the class of spaces with a  $\pi$ -tree [5, Remark 11]. In this paper, we build a space with a Lusin  $\pi$ -base whose square has no Lusin  $\pi$ -base (see Theorem 4.11).

The Baire space  $\omega^\omega$ , the Sorgenfrey line  $\mathcal{S}$ , and the irrational Sorgenfrey line  $\mathcal{I}$  have a Lusin  $\pi$ -base [4] and [6], and all at most countable products of  $\omega^\omega$ ,  $\mathcal{S}$ , and  $\mathcal{I}$  also have a Lusin  $\pi$ -base [6]. If a space  $X$  has a Lusin  $\pi$ -base, then the products  $X \times \omega^\omega$ ,  $X \times \mathcal{S}$ , and  $X \times \mathcal{S}^\omega$  have a Lusin  $\pi$ -base [6], and also  $X \setminus F$  has a Lusin  $\pi$ -base whenever  $F \subseteq X$  is a  $\sigma$ -compact [5] (but a dense open subset of  $X$  can be without a Lusin  $\pi$ -base).

If a space  $X$  has a Lusin  $\pi$ -base, then  $X$  can be mapped onto an arbitrary nonempty Polish space by a continuous open map and also  $X$  can be mapped onto  $\omega^\omega$  by a continuous one-to-one map [4]. If a space  $X$  has a Lusin  $\pi$ -base, then it has a countable  $\pi$ -base and a countable pseudo-base (both with clopen members); and also  $X$  is a Choquet space (but it can be not strong Choquet even in a separable metrizable case).

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