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TOPOLOGY PROCEEDINGS

Volume 60, 2022

Pages 31–44

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Electronically published on August 20, 2021

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
TOPOLOGY
PROCEEDINGS
Volume 60 (2022)
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E-Published on August 20, 2021

ANY NONTRIVIAL KNOT PROJECTION WITH NO TRIPLE CHORDS HAS A MONOGON OR A BIGON

NOBORU ITO AND YUSUKE TAKIMURA

Abstract. A generic immersion of a circle into a 2-sphere is often studied as a projection of a knot; it is called a knot projection. A chord diagram is a configuration of paired points on a circle; traditionally, the two points of each pair are connected by a chord. A triple chord is a chord diagram consisting of three chords, each of which intersects the other chords. Every knot projection obtains a chord diagram in which every pair of points corresponds to the inverse image of a double point. In this paper, we show that for any knot projection P, if its chord diagram contains no triple chord, then there exists a finite sequence from P to a simple closed curve such that the sequence consists of flat Reidemeister moves, each of which decreases 1-gons or strong 2-gons, where a strong 2-gon is a 2-gon oriented by an orientation of P.

1. Introduction

V. I. Arnold [1] (V. A. Vassiliev [16], respectively) introduces a theory classifying plane curves (knots, respectively) in vector spaces generated by immersions $S^1 \to \mathbb{R}^2$ ($\mathbb{R}^3$, respectively) divided by subspaces, called discriminants, each of which consists of curves (knots, respectively) with singularities. For the knot case, it is well known that every coefficient in the Taylor expansion $t = e^x$ of the Jones polynomial is a Vassiliev invariant [2]. For plane curves, Arnold [1, p. 16, Remark] writes,

2020 *Mathematics Subject Classification.* Primary: 57K10, 57K12.
Key words and phrases. knot projections; Reidemeister moves.
The part of the work of N. Ito was supported by JSPS Japanese-German Graduate Externship, a Waseda University Grant for Special Research Projects (Project number: 2015K-342), and MEXT KAKENHI Grant Number 20K03604. N. Ito was a project researcher of Grant-in-Aid for Scientific Research (S) 24224002.