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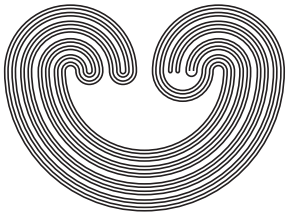

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by

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PROPERTIES OF MULTI-VALUED FUNCTIONS

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Abstract. For a given topological property $\mathcal{P}$ of mappings, we say that a multi-valued function $f : X \Rightarrow Y$ is dir-$\mathcal{P}$ (co-$\mathcal{P}$) if the projection from the graph of f to Y (onto X) has property $\mathcal{P}$. In this article, we investigate properties of dir-$\mathcal{P}$ and co-$\mathcal{P}$ functions. We discuss composition property, composition factor property, and product property of such functions.

1. Introduction

Since 2006, when W. T. Ingram and William S. Mahavier [8] introduced the notion of inverse limits with set-valued upper semi-continuous functions, the subject of set-valued functions has become quite popular. Some authors use such functions with additional properties concerning projections from the graph of the function to either domain or co-domain with special properties, like monotonicity, openness, confluence, weak confluence, lightness, or atomicity. Monotone projections are considered in [1] and [10], atomic projections are considered in [4], weakly confluent projections in [10], and open and confluence projections in [5], to mention only a few.

The aim of this article is to gather basic properties of multi-valued functions whose projections possess some properties. We call such functions dir-$\mathcal{P}$ or co-$\mathcal{P}$, where $\mathcal{P}$ is a given topological property. We investigate composition property, composition factor property, and product property of such functions. In section 4, we prove some other results about previously considered functions.

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† Sadly, Professor Włodzimierz J. Charatonik passed away in 2020.