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by

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# CONNECTED GENERALIZED INVERSE LIMITS AND INTERMEDIATE VALUE PROPERTY

TAVISH J. DUNN

Abstract. In this paper, we consider inverse limits of $[0,1]$ using upper semicontinuous set-valued functions. We introduce two generalizations of the intermediate value property and prove that inverse limits with upper semicontinuous set-valued bonding functions are connected if the bonding functions are surjective, have connected graphs, and have either generalization of the intermediate value property. Examples are given to demonstrate that if any of the conditions is dropped, the result does not hold in general. An example is given to show that an inverse limit may be connected even if the bonding functions do not have either intermediate value property. Further, we compare the structures of set-valued functions with the two types of the intermediate value property.

## 1. Introduction

Inverse limits have been objects of study for decades, for both continua and dynamical systems. William S. Mahavier [8] and W. T. Ingram and Mahavier [6] introduce the concept of inverse limits with upper semicontinuous (usc) set-valued functions. Since then, extensive work has been done to determine conditions under which various results for classical inverse limits can be generalized to inverse limits with usc functions.

One property of classical inverse limits that does not hold, in general, for inverse limits of usc functions is that the inverse limit of continua is a continuum. Ingram and Mahavier [6] find that even inverse limits of usc functions on $[0,1]$ are compact and nonempty but not necessarily connected. Ingram presents the following problems in [5]:

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