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ON BROUWER'S FIXED POINT THEOREM

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Abstract. It is shown by Klaas Pieter Hart, Jan van Mill, and Roman Pol [*Remarks on hereditarily indecomposable continua* Topology Proc. 25 (2000), 179–206] that Brouwer's fixed point theorem can be reduced to its 3-dimensional case by using the hyperspace of a 2-dimensional hereditarily indecomposable continuum. In this paper, we give a more direct and geometric argument that reduces the fixed point theorem to its 3-dimensional version.

1. Introduction

Brouwer's fixed point theorem is probably the most well-known result in topology. It states that a continuous map

$$f\colon I^n \to I^n$$

from an n-dimensional cube to itself has at least one fixed point: $f(x_0) = x_0$. Being equivalent to the no-retraction theorem and the statement that the topological dimension of I^n is equal to n, this theorem lies at the core of algebraic topology and dimension theory. It also has various generalizations and applications to other fields, including the theory of differential equations, symplectic geometry, physics, and game theory (one outstanding example is the Nash equilibrium theorem for finite non-cooperative games). For a good overview on what is classically known about Brouwer's fixed point theorem and its generalizations, we refer the reader to [9]; for Brouwer's biography, see [1].

More recently, it has been shown by Klaas Pieter Hart, Jan van Mill, and Roman Pol [4] that Brouwer's fixed point theorem can be reduced

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