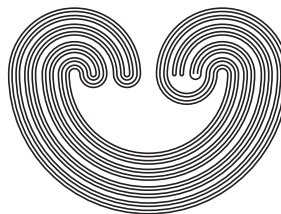


<http://topology.nipissingu.ca/tp/>

TOPOLOGY PROCEEDINGS



Volume 60, 2022

Pages 137–145

THREE PROBLEMS IN CONVERGENCE THEORY

by

JERZY WOJCIECHOWSKI

Electronically published on March 18, 2022

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.

See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.nipissingu.ca/tp/>

Mail: Topology Proceedings

Department of Mathematics & Statistics

Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



THREE PROBLEMS IN CONVERGENCE THEORY

JERZY WOJCIECHOWSKI

ABSTRACT. In this note, we prove that the class of paratopologies is simple and that under the assumption that the measurable cardinals form a proper class, the class of hypotopologies is not simple. Moreover, we provide an example of a Hausdorff convergence with idempotent set adherence (subdiagonal convergence) that is not weakly diagonal.

1. INTRODUCTION

One way to describe a topological space is to consider the neighborhood filters of points and the convergence relation between points and filters defined using the neighborhood filters. Convergence theory studies this relation in greater generality and considers the topological convergence only as a special case. The need to study non-topological convergences is pointed out by G. Choquet in his fundamental paper [5], where he investigates natural convergences on the family of closed subsets of a topological space and concludes that some of them are not topological unless the underlying topology is locally compact.

The exact collection of axioms required for a convergence space to satisfy varies in the literature. We follow the definition of Szymon Dolecki in [9] (see also [11] and [10]). A *convergence* ξ on a nonempty set X is a relation between the elements of X and the filters on X . Given a filter \mathcal{F} on X and $x \in X$, we write $x \in \lim_{\xi} \mathcal{F}$ when $(x, \mathcal{F}) \in \xi$, and we require that $\lim_{\xi} \mathcal{F} \subseteq \lim_{\xi} \mathcal{G}$ whenever $\mathcal{F} \subseteq \mathcal{G}$ and that $x \in \lim_{\xi} \{x\}^{\uparrow}$ for every $x \in X$, where $\{x\}^{\uparrow} := \{A \subseteq X : x \in A\}$ is the principal ultrafilter generated by x . In particular, any topology on a set X induces a convergence τ defined

2020 *Mathematics Subject Classification.* 54A20.

Key words and phrases. convergence spaces, diagonal properties of convergence spaces, hypotopologies, initially dense convergences, paratopologies.

©2022 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers.
See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.