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THREE PROBLEMS IN CONVERGENCE THEORY

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Abstract. In this note, we prove that the class of paratopologies is simple and that under the assumption that the measurable cardinals form a proper class, the class of hypotopologies is not simple. Moreover, we provide an example of a Hausdorff convergence with idempotent set adherence (subdiagonal convergence) that is not weakly diagonal.

1. Introduction

One way to describe a topological space is to consider the neighborhood filters of points and the convergence relation between points and filters defined using the neighborhood filters. Convergence theory studies this relation in greater generality and considers the topological convergence only as a special case. The need to study non-topological convergences is pointed out by G. Choquet in his fundamental paper [5], where he investigates natural convergences on the family of closed subsets of a topological space and concludes that some of them are not topological unless the underlying topology is locally compact.

The exact collection of axioms required for a convergence space to satisfy varies in the literature. We follow the definition of Szymon Dolecki in [9] (see also [11] and [10]). A *convergence* ξ on a nonempty set X is a relation between the elements of X and the filters on X. Given a filter $\mathcal{F}$ on X and $x \in X$, we write $x \in \lim_\xi \mathcal{F}$ when $(x, \mathcal{F}) \in \xi$, and we require that $\lim_\xi \mathcal{F} \subseteq \lim_\xi \mathcal{G}$ whenever $\mathcal{F} \subseteq \mathcal{G}$ and that $x \in \lim_\xi \{x\}^\uparrow$ for every $x \in X$, where $\{x\}^\uparrow := \{A \subseteq X : x \in A\}$ is the principal ultrafilter generated by x. In particular, any topology on a set X induces a convergence τ defined

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