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by

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# THERE IS A KATĚTOV SPACE THAT IS NOT COUNTABLY PARACOMPACT

MAKOTO KUROSAKI

Abstract. We construct, in ZFC, a Katětov space that is not countably paracompact. This answers a question of M. Katětov [Extension of locally finite coverings (Russian), Colloq. Math. 6 (1958), 145–151] and Teodor C. Przymusiński and Michael L. Wage [Collectionwise normality and extensions of locally finite coverings, Fund. Math. 109 (1980), no. 3, 175–187].

## 1. Introduction

In 1980, Teodor C. Przymusiński and Michael L. Wage [7] defined a space $X$ to be *Katětov* if $X$ is normal and every locally finite open cover of a closed subspace of $X$ can be extended to a locally finite open cover of $X$; i.e., for every locally finite open cover $\mathcal{U}$ of a closed subspace $A$ of $X$, there exists a locally finite open cover $\{V_U : U \in \mathcal{U}\}$ of $X$ such that $V_U \cap A = U$ for each $U \in \mathcal{U}$. Before that, in 1958, this notion was studied by M. Katětov [6] who asked whether every Katětov space is countably paracompact. A space $X$ is called *countably paracompact* if every countable open cover of $X$ has a locally finite open refinement. Przymusiński and Wage in [7] constructed a Katětov space that is not countably paracompact under the assumption of V = L and asked whether such a space can be constructed from only ZFC. Later, Zoltan T. Balogh, in [1], invented a flexible technique for constructing Dowker spaces in ZFC. A space $X$ is called *Dowker* if $X$ is normal but not countably paracompact. In this paper, using this technique, we construct a Katětov space that is not countably paracompact in ZFC.

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