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THE INVERSE LIMIT PROPERTY FOR SUBCONTINUA OF INVERSE LIMITS OF SET-VALUED FUNCTIONS

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ABSTRACT. We examine when a subcontinuum of an inverse limit with set-valued functions is equal to the inverse limit of its projections. There are multiple ways to interpret this problem. We focus on one particular interpretation due to Iztok Banič et al. [Bull. Malays. Math. Sci. Soc. 42 (2017), no. 3, 835–846]. When this does hold, we say that the inverse sequence has $\mathrm{ILP}(C(\varprojlim f_i))$. We identify properties of the bonding functions which imply that the system has $\mathrm{ILP}(C(\varprojlim f_i))$. We also show a relationship between this property and connectedness im kleinen and use this to identify inverse sequences that do not have $\mathrm{ILP}(C(\varprojlim f_i))$.

1. Introduction

Inverse limits of continuous, single-valued functions have been studied in the context of continuum theory for many decades. In 2004, William S. Mahavier [9] introduced the concept of inverse limits of closed subsets of $[0, 1]$, and this was then extended by W. T. Ingram and Mahavier [6] to inverse limits of set-valued functions on compact Hausdorff spaces. Since that time, there have been many articles written on the topic with a great deal of focus on taking results from inverse limits of single-valued functions and finding the proper generalization to the context of set-valued functions. Ingram provides a thorough introduction to this topic in [4].

One well-known result for inverse limits of single-valued functions is the closed subset theorem. This states that if $\{X_i, f_i\}_{i=1}^{\infty}$ is an inverse sequence where each $f_i\colon X_{i+1} \to X_i$ is continuous, then for every closed

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