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by

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# IMAGE PARTITION REGULAR MATRICES AND CONCEPTS OF LARGENESS, II

NEIL HINDMAN AND DONA STRAUSS

Abstract. Let $u$ and $v$ be positive integers and let $A$ be a $u \times v$ matrix with rational entries. We determine several characterizations of the property that whenever $B$ is a piecewise syndetic subset of the set $\mathbb{N}$ of positive integers, $\{\vec{x} \in \mathbb{N}^v : A\vec{x} \in B^u\}$ is piecewise syndetic in $\mathbb{N}^v$ as well as the corresponding property with $\mathbb{Z}$ replacing $\mathbb{N}$. We investigate related phenomena for several other notions of largeness in a semigroup.

## 1. Introduction

We are concerned throughout with the notion of *image partition regularity* of a matrix.

**Definition 1.1.** Let $u, v \in \mathbb{N}$, and let $A$ be a $u \times v$ matrix with rational entries.

(1) The matrix $A$ is *image partition regular over* $\mathbb{N}$ (IPR/$\mathbb{N}$) if and only if, whenever $\mathbb{N}$ is finitely colored, there exists $\vec{x} \in \mathbb{N}^v$ such that the entries of $A\vec{x}$ are monochromatic.
(2) The matrix $A$ is *image partition regular over* $\mathbb{Z}$ (IPR/$\mathbb{Z}$) if and only if, whenever $\mathbb{Z} \setminus \{0\}$ is finitely colored, there exists $\vec{x} \in (\mathbb{Z} \setminus \{0\})^v$ such that the entries of $A\vec{x}$ are monochromatic.

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