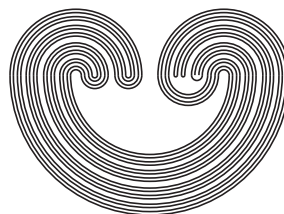


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## IMAGE PARTITION REGULAR MATRICES AND CONCEPTS OF LARGENESS, II

by

NEIL HINDMAN AND DONA STRAUSS

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## IMAGE PARTITION REGULAR MATRICES AND CONCEPTS OF LARGENESS, II

NEIL HINDMAN AND DONA STRAUSS

**ABSTRACT.** Let  $u$  and  $v$  be positive integers and let  $A$  be a  $u \times v$  matrix with rational entries. We determine several characterizations of the property that whenever  $B$  is a piecewise syndetic subset of the set  $\mathbb{N}$  of positive integers,  $\{\vec{x} \in \mathbb{N}^v : A\vec{x} \in B^u\}$  is piecewise syndetic in  $\mathbb{N}^v$  as well as the corresponding property with  $\mathbb{Z}$  replacing  $\mathbb{N}$ . We investigate related phenomena for several other notions of largeness in a semigroup.

### 1. INTRODUCTION

We are concerned throughout with the notion of *image partition regularity* of a matrix.

**Definition 1.1.** Let  $u, v \in \mathbb{N}$ , and let  $A$  be a  $u \times v$  matrix with rational entries.

- (1) The matrix  $A$  is *image partition regular over  $\mathbb{N}$*  (IPR/ $\mathbb{N}$ ) if and only if, whenever  $\mathbb{N}$  is finitely colored, there exists  $\vec{x} \in \mathbb{N}^v$  such that the entries of  $A\vec{x}$  are monochromatic.
- (2) The matrix  $A$  is *image partition regular over  $\mathbb{Z}$*  (IPR/ $\mathbb{Z}$ ) if and only if, whenever  $\mathbb{Z} \setminus \{0\}$  is finitely colored, there exists  $\vec{x} \in (\mathbb{Z} \setminus \{0\})^v$  such that the entries of  $A\vec{x}$  are monochromatic.

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*Key words and phrases.* Stone-Čech compactification, image partition regular, semigroups, notions of size.

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