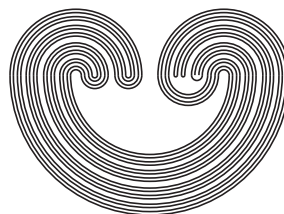


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by

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CHAINS IN PARTIALLY ORDERED SPACES

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ABSTRACT. In this paper we study chains in partially ordered spaces, topological spaces equipped with a partial order, which we assume to be semiclosed, i.e., $\downarrow x$ and $\uparrow x$ are closed for each x . The strong order-theoretic property of being a chain allows us to develop some significant theory about chains in the weak setting of a semiclosed relation. We obtain conditions for chains to be compact, to be connected, and to have the relative topology agree with the order topology. We illustrate applications of these results in the setting of semitopological semilattices.

1. INTRODUCTION

By a chain we mean a non-empty set equipped with a partial order that is a total order (also called a linear order). In this paper we consider quite general topological spaces equipped with partial orders, postulate a weak interaction between the two, and seek to see what can be said about subsets that are chains equipped with the relative topology. The property of being a chain is of course a strong hypothesis, and it is this fact that allows one to be able to develop some significant theory in an otherwise quite general setting. Chains in ordered topological spaces have been treated elsewhere and our material overlaps those treatments; see, for example, [12], [2], and [4, Section VI-5].

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