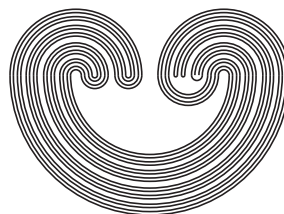


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## THE HALF DISC TOPOLOGY IS PARTIALLY METRIZABLE

by

SAMER ASSAF AND TOM CUCHTA

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## THE HALF DISC TOPOLOGY IS PARTIALLY METRIZABLE

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**ABSTRACT.** In this paper we answer the long standing conjecture: “Is a Hausdorff partial metric space metrizable?”, albeit in a negative way. To accomplish this, we show how to partially metrize the Half Disc Topological Space, a non-metrizable Completely Hausdorff space. We end by suggesting a new conjecture to replace the one we disproved.

### 1. INTRODUCTION

In 1994, Matthews [8] relaxed the metric axioms to obtain the partial metric axioms. We present the generalized axioms below as presented by O’Neill [10] since there is no need to restrict the partial metric values to non-negatives.

**Definition 1.1.** A partial metric on a set  $X$  is a function  $p: X \times X \rightarrow \mathbb{R}$  with the following properties for all  $x, y, z \in X$ :

( $p$ -lbd):  $p(x, x) \leq p(x, y)$

( $p$ -sym):  $p(x, y) = p(y, x)$

( $p$ -sep):  $p(x, x) = p(x, y) = p(y, y)$  if and only if  $x = y$

( $p$ -inq):  $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$

Just like in the metric case, we can use the partial metric to create bases consisting of  $p$ -open balls as presented by O’Neill [10].

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