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TOPOLOGY PROCEEDINGS

Volume 61, 2023

Pages 139–144

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by

Samer Assaf and Tom Cuchta

Electronically published on February 27, 2022

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
TOPOLOGY PROCEEDINGS
Volume 61 (2023)
Pages 139-144
E-Published on February 27, 2022

THE HALF DISC TOPOLOGY IS PARTIALLY METRIZABLE

SAMER ASSAF AND TOM CUCHTA

Abstract. In this paper we answer the long standing conjecture: "Is a Hausdorff partial metric space metrizable?", albeit in a negative way. To accomplish this, we show how to partially metrize the Half Disc Topological Space, a non-metrizable Completely Hausdorff space. We end by suggesting a new conjecture to replace the one we disproved.

1. Introduction

In 1994, Matthews [8] relaxed the metric axioms to obtain the partial metric axioms. We present the generalized axioms below as presented by O'Neill [10] since there is no need to restrict the partial metric values to non-negatives.

Definition 1.1. A partial metric on a set X is a function $p\colon X \times X \to \mathbb{R}$ with the following properties for all $x, y, z \in X$:
(p-lbnd): $p(x,x) \leq p(x,y)$
(p-sym): $p(x,y) = p(y,x)$
(p-sep): $p(x,x) = p(x,y) = p(y,y)$ if and only if $x = y$
(p-inq): $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$

Just like in the metric case, we can use the partial metric to create bases consisting of p-open balls as presented by O'Neill [10].

2020 *Mathematics Subject Classification.* Primary 54D10; Secondary 54E35, 54D15.
Key words and phrases. partial metric, separation axiom, half disc topology, completely hausdorff.