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by

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COMPLEMENTS OF TOPOLOGIES WITH SHORT SPECIALIZATION QUASIORDERS

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Abstract. By identifying a topology τ on a finite set X with its specialization quasiorder $\leq$, we investigate the complements of τ in the lattice of topologies on X in cases where the heights of the specialization posets are small.

Dedicated to Ralph Kopperman, a pioneer in asymmetric topology.

1. Introduction

If τ is a topology on a finite set X, associating the specialization quasiorder defined by $x \leq y$ if and only if $x \in cl\{y\}$ gives a one-to-one correspondence between the topologies and quasiorders on X. We interchangeably denote a topology τ by its specialization quasiorder $\leq$. Thus, the smallest neighborhood $N(x)$ of x corresponds to $\uparrow x = \{y \in X : x \leq y\}$, and we will use $N(x)$ and $\uparrow x$ interchangeably. A quasiorder $\leq$ on X gives an equivalence relation $\approx$ defined by $x \approx y$ if and only if $x \leq y$ and $y \leq x$, and gives a partial order on the $\approx$-equivalence classes defined $[x] \leq' [y]$ if and only if $x \leq y$. To draw the Hasse diagram for a quasiorder, we draw the Hasse diagram for the related partial order $\leq'$ and represent each point $[x] \in X/\approx$ by the set of points of X which make up $[x]$. In this context, we will call the set of points of $[x]$ a *cloud*. A quasiorder $\leq$ is a partial order if and only if each cloud is a singleton, or equivalently, if τ is T_0. A quasiorder $\leq$ is a total order if and only if τ is an irreducible T_0, T_5 topological space (see [6]) or equivalently, if τ is T_0 and is a nested collection of open sets. The height of a quasiordered set $(X, \leq)$ is the

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