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by

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CLOSURE OPERATORS ASSOCIATED WITH RELATIONS

JOSEF ŠLAPAL

Abstract. We study closure operators that are associated with α-ary relations where $\alpha > 1$ is an ordinal. In particular, we show that the connectedness with respect to the closure operators is a certain kind of path-wise connectedness. We demonstrate a possible application of our results in digital topology.

1. Introduction

It is always interesting and useful to study relationships between different mathematical structures. In this note, we focus on relationships between relations (with arities being positive ordinals) and closure operators obtained from the Kuratowski ones by omitting the requirements of additivity and idempotency (but keeping that of monotonicity). We will study closure operators associated with relations and, as the main result, it will be shown that the connectedness with respect to such closure operators corresponds to a pathwise connectedness with respect to the given relations. It is well known that closure operators have numerous applications, particularly in computer science [11]. We will discuss an application of our results in digital topology and will show that the closure operators associated with relations provide a convenient structure on the digital plane, which has some advantages over the Khalimsky topology dealt with in [5].

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Key words and phrases. Closure operator, ordinal (number), relation, path, connectedness, Khalimsky topology.

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