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A CATEGORICAL REVIEW OF COMPLETE REGULARITY

AMIR HOMAYOUN NEJAH AND WALTER THOLEN

Dedicated to the memory of Ralph Kopperman

Abstract. We use the ultrafilter-convergence axiomatics for topological spaces to motivate in detail a gentle categorical introduction, first to Barr's **Set**-based relational T-algebras, and then to Burroni's T-preorders internal to a category $\mathcal{C}$, here called T-spaces in $\mathcal{C}$, for a monad T on $\mathcal{C}$ that substitutes the ultrafilter monad on **Set**. Within these settings one finds not only the notions of compactness and Hausdorff separation, originally due to Manes, but also that of complete regularity. Based on a somewhat hidden result by Burroni, the main theorem of this paper establishes an external fibrational characterization of the category of completely regular T-spaces with its reflexive subcategory of compact Hausdorff T-spaces, under modest assumptions on $\mathcal{C}$ and T.

1. Introduction

The axiomatization of convergence was a key motivator for the development of the notions of metric and topology, as pursued primarily by Fréchet and Hausdorff in the early twentieth century. The 1967 thesis by Manes [22] (see also [23]) characterized compact Hausdorff spaces as the

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Key words and phrases. ultrafilter convergence; completely regular space; T-space; monotone map; topological category; Stone-Čech compactification; reflective subcategory; Grothendieck fibration.

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