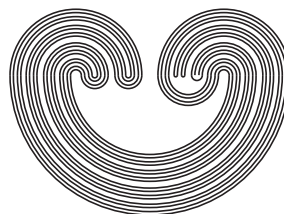

TOPOLOGY PROCEEDINGS

Volume 61, 2023

Pages 305–340



A CATEGORICAL REVIEW OF COMPLETE REGULARITY

by

AMIR HOMAYOUN NEJAH AND WALTER THOLEN

Electronically published on January 14, 2023

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.

Topology Proceedings

Web: <http://topology.nipissingu.ca/tp/>

Mail: Topology Proceedings
Department of Mathematics & Statistics
Auburn University, Alabama 36849, USA

E-mail: topolog@auburn.edu

ISSN: (Online) 2331-1290, (Print) 0146-4124

COPYRIGHT © by Topology Proceedings. All rights reserved.



A CATEGORICAL REVIEW OF COMPLETE REGULARITY

AMIR HOMAYOUN NEJAH AND WALTER THOLEN

Dedicated to the memory of Ralph Kopperman

ABSTRACT. We use the ultrafilter-convergence axiomatics for topological spaces to motivate in detail a gentle categorical introduction, first to Barr's **Set**-based relational T -algebras, and then to Burroni's T -preorders internal to a category \mathcal{C} , here called T -spaces in \mathcal{C} , for a monad T on \mathcal{C} that substitutes the ultrafilter monad on **Set**. Within these settings one finds not only the notions of compactness and Hausdorff separation, originally due to Manes, but also that of complete regularity. Based on a somewhat hidden result by Burroni, the main theorem of this paper establishes an external fibrational characterization of the category of completely regular T -spaces with its reflexive subcategory of compact Hausdorff T -spaces, under modest assumptions on \mathcal{C} and T .

1. INTRODUCTION

The axiomatization of convergence was a key motivator for the development of the notions of metric and topology, as pursued primarily by Fréchet and Hausdorff in the early twentieth century. The 1967 thesis by Manes [22] (see also [23]) characterized compact Hausdorff spaces as the

2020 *Mathematics Subject Classification.* 54D15, 54D30, 54B30, 18A40, 18C20, 18D30, 18F60.

Key words and phrases. ultrafilter convergence; completely regular space; T -space; monotone map; topological category; Stone-Čech compactification; reflective subcategory; Grothendieck fibration.

The second author acknowledges the support received under the Discovery Grants Program (no. 501260) of the Natural Sciences and Engineering Council of Canada. This grant and the assistance of the Fields Institute for the Mathematical Sciences supported the stay of the first author at York University in 2021-22, during which this work was completed.

©2023 Topology Proceedings.

This file contains only the first page of the paper. The full version of the paper is available to Topology Proceedings subscribers. See <http://topology.nipissingu.ca/tp/subscriptioninfo.html> for information.