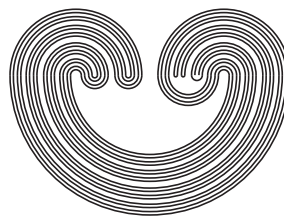


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NOTES ON QUASI-METRIZABILITY AND TREES

by

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NOTES ON QUASI-METRIZABILITY AND TREES

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ABSTRACT. Without trying to be comprehensive, this article introduces some necessary and sufficient conditions under which a tree is quasi-metrizable. Sufficient conditions include the concepts of \mathbb{Q} -special and \mathbb{R} -special, and necessary ones include height $\leq \omega_1$ and lack of uncountable branches. It is shown how a Souslin tree is not quasi-metrizable by using a forcing and absoluteness argument, and the same is done for a tree defined in ZFC.

1. INTRODUCTION

A quasi-metric is a ‘distance function’ like a metric, but without assuming symmetry.

Definition 1.1. A *quasi-metric* on a set X is a function $d : X \times X \rightarrow \mathbb{R}^+ \cup \{0\}$ such that, for each $x, y, z \in X$,

- (i) $d(x, y) = 0$ iff $x = y$ and
- (ii) $d(x, z) \leq d(x, y) + d(y, z)$.

The order in (ii) is important, unlike in a metric space, where it is harmless to write $d(z, y)$ at the end of the triangle inequality. Order is also important in defining $B(x, \epsilon)$, the ϵ -ball centered on x . It is customarily defined as $\{y : d(x, y) < \epsilon\}$.

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