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TOPOLOGY PROCEEDINGS

Volume 62, 2023

Pages 1–12

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Electronically published on September 20, 2022

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
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Volume 62 (2023)
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E-Published on September 20, 2022

CONNECTED SPACES AND FORCING

AKIRA IWASA

Abstract. We study how forcing destroys connectedness of topological spaces. A topological space is called strongly connected if every continuous real-valued function on it is constant. We give an example where forcing destroys strong connectedness of a space.

1. Introduction

Let $\mathbf{V}$ be a ground model. For a forcing $\mathbb{P}$, let $\mathbf{V}^{\mathbb{P}}$ denote the forcing extension of $\mathbf{V}$ by $\mathbb{P}$. For a topological space $(X, \mathcal{T}_X)$ in $\mathbf{V}$, we consider a topological space $(X, \mathcal{T}_X^{\mathbb{P}})$ in $\mathbf{V}^{\mathbb{P}}$ such that $\mathcal{T}_X^{\mathbb{P}}$ is the topology generated by $\mathcal{T}_X$ in $\mathbf{V}^{\mathbb{P}}$. By definition, $\mathcal{T}_X$ is a base for $\mathcal{T}_X^{\mathbb{P}}$. For a topological property φ, we say that a forcing $\mathbb{P}$ preserves φ, provided whenever $(X, \mathcal{T}_X)$ has property φ, $(X, \mathcal{T}_X^{\mathbb{P}})$ also has property φ. If a forcing $\mathbb{P}$ does not preserve property φ, then we say that $\mathbb{P}$ destroys property φ.

In [4], Renata Grunberg, L'ucia R. Junqueira, and Franklin D. Tall studied conditions under which normality is preserved by adding Cohen reals. In [3], William G. Fleissner, Tim LaBerge, and Adrienne Stanley invented a machine that turns a Dowker space into another Dowker space whose normality is destroyed by adding a Cohen real. In [5], we studied how forcing destroys Fréchet-Urysohn property. The purpose of this note is to study how forcing destroys connectedness of topological spaces. Let us give definitions.

Definition 1.1. A topological space is said to be *connected* if the empty set and the whole space are the only clopen (= closed and open) subsets of the space.

2020 *Mathematics Subject Classification.* Primary 54A35, 54D05.
Key words and phrases. connected, forcing.