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METRIZABILITY OF MAHAVIER PRODUCTS INDEXED BY PARTIAL ORDERS

STEVEN CLONTZ AND JACOB DUNHAM

Abstract. Let X be separable metrizable, and let $f \subseteq X^2$ be a non-trivial relation on X. For a given partial order $\langle P, \leq \rangle$, the Mahavier product $\mathbf{M}\langle X, f, P\rangle \subseteq X^P$ (also known as a generalized inverse limit) collects functions such that $x(p) \in f(x(q))$ for all $p \leq q$. Steven Clontz and Scott Varagona [Topology Proc. 54 (2019), 259–269] previously showed that for well orders P, $\mathbf{M}\langle X, f, P\rangle$ is separable metrizable exactly when P is countable and f satisfies condition Γ; we extend this result to hold for all partial orders.

1. Introduction

Let X be a separable metrizable topological space, let $f \subseteq X^2$ be a relation on X, and let Q be a set preordered by (reflexive and transitive) $\preceq$.

Extending work done in e.g., [5] and [1], we consider the subspace $\mathbf{M}\langle X, f, Q\rangle$ of X^Q where $x(p) \in f(x(q))$ for all $p \preceq q$. Such subspaces are often known as generalized inverse limits or, as we will refer to them, Mahavier products. For an introduction to such structures, often considered in the context of continuum theory, we direct the reader to [6]. Much of the literature on this subject considers only simple indices, particularly $Q = \mathbb{N}$ or $\mathbb{Z}$ with its usual order.

It is immediate that any subspace of X^Q is separable metrizable whenever Q is countable. In [2] and [3], the authors consider whether $\mathbf{M}\langle X, f, Q\rangle$ might be metrizable when Q is an uncountable well-order. It turns out that, except in trivial situations, the answer is no.

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