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INVERSE LIMITS WITH SMITH FUNCTIONS

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Abstract. We call an upper semi-continuous function $f : [0,1] \to 2^{[0,1]}$ a *Smith function* if f is surjective, the graph of f is connected, and the graph of f is the union of finitely many vertical and horizontal line segments. We introduced the definition of a Smith function (named in honor of Michel Smith) at the 2021 Spring Topology and Dynamical Systems Conference. In this paper, we present various theorems about inverse limits whose bonding functions are Smith functions. We focus on the connectedness and indecomposability of such inverse limits, but some other topics are discussed as well. Numerous examples and open problems are also provided.

1. Introduction

In 2006, W. T. Ingram and William S. Mahavier laid the foundation for the study of inverse limits with upper semi-continuous (u.s.c.) bonding functions in a landmark paper [12]. Since then, the theory of generalized inverse limits has been ever-expanding, with new results appearing in the literature every year.

The graph of a u.s.c. function on $[0,1]$ can be obtained by drawing any closed subset K of $[0,1] \times [0,1]$ such that $\pi_1(K) = [0,1]$. We therefore have much more freedom when producing examples of u.s.c. inverse limits than we previously had with traditional inverse limits. However, this extra freedom can make it more difficult to prove theorems about generalized inverse limits, since there are many "ill-behaved" u.s.c. functions that a potential theorem would have to handle. So researchers often restrict their attention to certain well-behaved types of u.s.c. bonding functions—for example, continuum-valued u.s.c. functions, or u.s.c. functions whose

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