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by

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PRISM COMPLEXES

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Abstract. A prism is the product space $\Delta \times I$ where Δ is a 2-simplex and I is a closed interval. We introduce prism complexes as an analogue of simplicial complexes and show that every compact 3-manifold has a prism complex structure. We call a prism complex *special* if each interior horizontal edge lies in four prisms, each boundary horizontal edge lies in two prisms, and no horizontal face lies on the boundary. We give a criterion for existence of horizontal surfaces in (possibly non-orientable) Seifert fiber spaces. Using this, we show that a compact 3-manifold admits a special prism complex structure if and only if it is a Seifert fiber space with non-empty boundary, a Seifert fiber space with a non-empty collection of surfaces in its exceptional set, or a closed Seifert fiber space with Euler number zero. So, in particular, a compact 3-manifold with boundary is a Seifert fiber space if and only if it has a special prism complex structure.

1. Introduction

A closed 3-manifold M is called *irreducible* if every embedded 2-sphere in M bounds a 3-ball. If M is reducible, then there exist finitely many disjointedly embedded 2-spheres in M such that after cutting M along these spheres and capping off the spherical boundaries with 3-balls, the closed manifolds obtained are either irreducible or $S^2 \times S^1$. These capped off pieces can be further decomposed along a canonical collection of disjointedly embedded tori into compact manifolds which have three possibilities: They are either finitely covered by torus bundles, or they are Seifert fiber

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