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by

SALOUA CHOUINGOU AND ABDELHADI ZAIM

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ON THE RANK OF MAPS INDUCED BY FIBRATIONS IN HOMOTOPY AND HOMOLOGY

SALOUA CHOUINGOU AND ABDELHADI ZAIM

ABSTRACT. Let $F \to E \xrightarrow{p} B$ be a fibration of simply connected elliptic CW-complexes. Denote Im $\pi_*(p) \otimes \mathbb{Q}$ by

$$\bigoplus_{i} \operatorname{Im} \left\{ \pi_i(p) \otimes \mathbb{Q} : \pi_i(E) \otimes \mathbb{Q} \to \pi_i(B) \otimes \mathbb{Q} \right\}$$

and Im $H_*(p;\mathbb{Q})$ by

$$\bigoplus_{i} \operatorname{Im} \left\{ H_i(p; \mathbb{Q}) : H_i(E; \mathbb{Q}) \to H_i(B; \mathbb{Q}) \right\}.$$

The topological aspect of this paper is centered around answering the question

Is rank
$$\pi_*(p) \otimes \mathbb{Q} \leq \operatorname{rank} H_*(p;\mathbb{Q})$$
?

We are able to prove that, in general, the response must be negative, but in this paper, we will prove the positive in certain reasonable cases.

1. Introduction

Let X be a simply-connected CW-complex of finite type over \mathbb{Q} , i.e., have finite dimensional rational cohomology in each degree. We say that X is rationally elliptic if the dimensions of cohomology and homotopy are both finite, i.e.,

$$\dim \pi_*(X) \otimes \mathbb{Q} < \infty$$
 and $\dim H^*(X;\mathbb{Q}) < \infty$.

The computation of rational cohomology and homotopy groups of elliptic spaces have been receiving growing attention and have become a popular subject of study with a lot of progress. In particular, for these groups, the famous Hilali conjecture states that the dimension of the rational

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 $Key\ words\ and\ phrases.$ elliptic spaces, Hilali conjecture, rational homotopy theory, Sullivan models, Yamaguchi–Yokura conjecture.

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