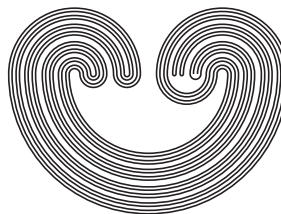


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## $C^*$ -EMBEDDED DENSE SUBSETS OF $z$ -NEIGHBORHOOD-SUBLINEAR SPACES ARE $P$ -EMBEDDED

by

YASUSHI HIRATA, NOBUYUKI KEMOTO, AND HARUTO OHTA

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Department of Mathematics & Statistics  
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**$C^*$ -EMBEDDED DENSE SUBSETS OF  
 $z$ -NEIGHBORHOOD-SUBLINEAR SPACES  
ARE  $P$ -EMBEDDED**

YASUSHI HIRATA, NOBUYUKI KEMOTO, AND HARUTO OHTA

**ABSTRACT.** We define a concept of  $z$ -neighborhood-sublinear space and point out that

- every first-countable Tychonoff space and every generalized ordered space is  $z$ -neighborhood-sublinear, and
- subspaces and finite products of  $z$ -neighborhood-sublinear spaces are  $z$ -neighborhood-sublinear.

As a main theorem, we prove that every  $C^*$ -embedded dense subset of a  $z$ -neighborhood-sublinear space is  $P$ -embedded.

The first author and Yukinobu Yajima, in [ $C^*$ -embedding implies  $P$ -embedding in products of ordinals, *Topology and Appl.*, 231 (2017), pp. 251–265], prove that for all subspaces  $A$  and  $B$  of an ordinal, if a closed subset  $F$  of  $A \times B$  is  $C^*$ -embedded in  $A \times B$ , then  $F$  is  $P$ -embedded in  $A \times B$ . We can remove closedness from the assumption by applying the main theorem.

## 1. INTRODUCTION

Let  $\mathbb{R}$  be the real line, and  $\mathbb{I}$  the unit interval, i.e.,  $\mathbb{I} = [0, 1] \subset \mathbb{R}$ . A subset  $E$  in a space  $X$  is said to be  $C^*$ -*embedded* ( $C$ -embedded, respectively) in  $X$  if every continuous function from  $E$  into  $\mathbb{I}$  ( $\mathbb{R}$ , respectively) is continuously extended over  $X$ . A subset  $E$  in  $X$  is said to be  $P$ -*embedded* in  $X$

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*Key words and phrases.*  $C^*$ -embedded, generalized ordered space, globular space, lob-space,  $P$ -embedded, product, subspace of an ordinal,  $z$ -neighborhood-sublinear space.

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