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TOPOLOGY PROCEEDINGS

Volume 62, 2023

Pages 117–131

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Electronically published on March 6, 2023

Topology Proceedings

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E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
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Volume 62 (2023)
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JEAN GOUBAULT-LARRECQ

Abstract. We show that the locally strongly sober spaces are exactly the coherent sober spaces that are weakly Hausdorff in the sense of Klaus Keimel and Jimmie D. Lawson. This allows us to describe their Stone duals explicitly. As another application, we show that weak Hausdorffness is a sufficient condition for lenses and of quasi-lenses to form homeomorphic spaces, generalizing previously known results.

1. Introduction

In their study of measure extension theorems for T_0 spaces, Klaus Keimel and Jimmie D. Lawson [14, Lemma 6.6] introduce so-called weakly Hausdorff spaces. The notion does not seem to have been investigated much. We will show that these spaces are related to the more well-known locally strongly sober spaces: In section 3, we show that the latter are exactly the weakly Hausdorff, coherent sober spaces. This allows us to elucidate their Stone duals in section 5 as those frames in which the filter-theoretic join of any two Scott-open filters is Scott-open. The notion of weakly Hausdorff spaces also helps us generalize a few results on variants of the so-called Plotkin powerdomain, namely on spaces of lenses and of quasi-lenses, with shorter proofs, as we will see in section 6. Our intermediate section 4 will be an opportunity of making remarks and giving examples and counterexamples. We start off immediately with section 2, giving the required preliminaries.

2020 *Mathematics Subject Classification.* Primary 54D10, 54A20, 54B20; Secondary 68Q55, 54H30, 06F30.

Key words and phrases. lens, locally strongly sober, Stone duality, temperate frame, weakly Hausdorff.