http://topology.nipissingu.ca/tp/

TOPOLOGY PROCEEDINGS

Volume 62, 2023

Pages 133–149

Characterizing Endpoints for a Family of Set-valued Inverse Limits

by

Lori Alvin, Katherine Betts, and James P. Kelly

Electronically published on March 30, 2023

Topology Proceedings

Web:	http://topology.nipissingu.ca/tp/
Mail:	Topology Proceedings Department of Mathematics & Statistics Auburn University, Alabama 36849, USA
E-mail:	topolog@auburn.edu
ISSN:	(Online) 2331-1290, (Print) 0146-4124

http://topology.nipissingu.ca/tp/
TOPOLOGY PROCEEDINGS
Volume 62 (2023)
Pages 133-148
E-Published on March 30, 2023

CHARACTERIZING ENDPOINTS FOR A FAMILY OF SET-VALUED INVERSE LIMITS

LORI ALVIN, KATHERINE BETTS, AND JAMES P. KELLY

Abstract. We investigate the endpoints of inverse limits of set-valued functions using A. Lelek's definition of an endpoint. We provide two characterizations for a point to be an endpoint of the inverse limit for a family of set-valued functions. The first characterization utilizes limit points of intersections of special arcs in the inverse limit, whereas the second characterization focuses on sequences of branch points. The paper concludes with several examples demonstrating how endpoints can be identified using finite approximations of the inverse limit.

1. Introduction

Given an upper semi-continuous set-valued function $F : [0,1] \to 2^{[0,1]}$, we are interested in understanding and identifying the topological structure and underlying properties of $\varprojlim F$. Endpoints of inverse limits have been well studied in both the classical and set-valued settings, and classifying the collection of endpoints for an inverse limit can be a nontrivial task. There are many definitions for what it means for a point to be an endpoint of a continuum. In [2, p. 660], R. H. Bing defines p as an *endpoint* of the continuum X if for any two subcontinua $H, K \subseteq X$, both containing p, either $H \subseteq K$ or $K \subseteq H$. Using Bing's definition, James P. Kelly [4, Theorem 1.2] is able to show that $\mathbf{p} = (p_0, p_1, \ldots)$ is an endpoint of $\varprojlim F$ provided that for infinitely many $n \in \mathbb{N}$, $(p_0, p_1, \cdots, p_n)$ is an endpoint of $G_n = \{\mathbf{x} = (x_i)_{i=0}^n \in \prod_{i=0}^n X \colon x_{i-1} \in F(x_i)\}$. Further, in the special case where each bonding function has its inverse equal to the

2020 *Mathematics Subject Classification.* Primary 54F17; Secondary 54F65, 37B45.

Key words and phrases. branch points, endpoints, inverse limits, set-valued functions.