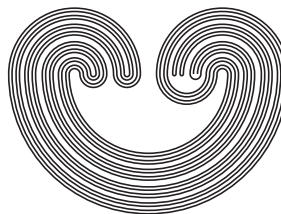


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by

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CHARACTERIZING ENDPOINTS FOR A FAMILY OF SET-VALUED INVERSE LIMITS

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ABSTRACT. We investigate the endpoints of inverse limits of set-valued functions using A. Lelek's definition of an endpoint. We provide two characterizations for a point to be an endpoint of the inverse limit for a family of set-valued functions. The first characterization utilizes limit points of intersections of special arcs in the inverse limit, whereas the second characterization focuses on sequences of branch points. The paper concludes with several examples demonstrating how endpoints can be identified using finite approximations of the inverse limit.

1. INTRODUCTION

Given an upper semi-continuous set-valued function $F : [0, 1] \rightarrow 2^{[0,1]}$, we are interested in understanding and identifying the topological structure and underlying properties of $\varprojlim F$. Endpoints of inverse limits have been well studied in both the classical and set-valued settings, and classifying the collection of endpoints for an inverse limit can be a nontrivial task. There are many definitions for what it means for a point to be an endpoint of a continuum. In [2, p. 660], R. H. Bing defines p as an *endpoint* of the continuum X if for any two subcontinua $H, K \subseteq X$, both containing p , either $H \subseteq K$ or $K \subseteq H$. Using Bing's definition, James P. Kelly [4, Theorem 1.2] is able to show that $\mathbf{p} = (p_0, p_1, \dots)$ is an endpoint of $\varprojlim F$ provided that for infinitely many $n \in \mathbb{N}$, (p_0, p_1, \dots, p_n) is an endpoint of $G_n = \{\mathbf{x} = (x_i)_{i=0}^n \in \prod_{i=0}^n X : x_{i-1} \in F(x_i)\}$. Further, in the special case where each bonding function has its inverse equal to the

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