

## Some Properties of Primal Topologies Seen as Semirings

by

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## SOME PROPERTIES OF PRIMAL TOPOLOGIES SEEN AS SEMIRINGS

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ABSTRACT. Given a set X and a function  $f: X \to X$ , a topology  $\tau_f$  is determined by taking the open sets to be those sets  $A \subset X$  such that  $f^{-1}(A) \subseteq A$ . The topological space  $(X, \tau_f)$  is called primal space or functional Alexandroff space. In this paper, we study some properties of primal topologies seen as semirings. We prove, for example, that if  $(X, \tau_f)$  is a connected primal space, then  $\tau_f$  is a local semiring. We also determine some topological conditions for a square matrix A to be invertible, considering the primal topology  $\tau_A$  generated by the matrix.

## 1. Introduction

In this article, we explore some properties of primal spaces, also known as functional Alexandroff spaces. These spaces can have important applications in mathematics, like those shown in [8], and in more diverse science areas, as seen in [5], [10], and [6]. In particular, the interest in the study of these topological spaces has accelerated since problems of great interest such as the Collatz conjecture can be expressed in topological terms, making particular use of primal spaces (see [3] and [11]) thus potentially pointing to its resolution.

These spaces were initially introduced in [9] by Fatemah Ayatollah Zadeh Shirazi and Nasser Golestani who show, among other results, the relationship between Alexandroff spaces and functional Alexandroff spaces. Then Othman Echi [1] refers to these spaces as primal spaces and

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